Many body localization and phase diagram of weakly interacting 1D bosons

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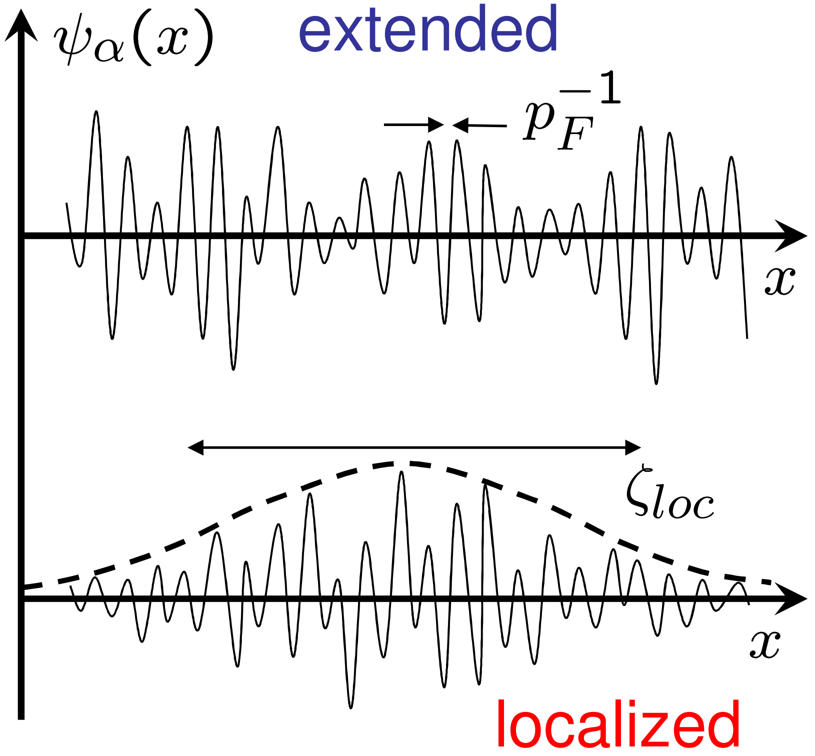
Outline:

- Remind: Many body localization and estimate for the transition temperature;
- Remind: Single particle localization in 1D;
- Remind: “Superconductor”-insulator transition at T=0;
- Many-body metal-insulator transiton at finite T;

1. Localization of single-electron wave-functions:

$$\left[-\frac{\nabla^2}{2m} + U(\boldsymbol{r}) - \epsilon_F\right]\psi_\alpha(\boldsymbol{r}) = \xi_\alpha\psi_\alpha(\boldsymbol{r})$$



d=1; All states are *localized*

Exact solution for one channel:

M.E. Gertsenshtein, V.B. Vasil'ev, (1959)

"Conjecture" for one channel:

Sir N.F. Mott and W.D. Twose (1961)

Exact solution for $\sigma(\omega)$ for one channel:

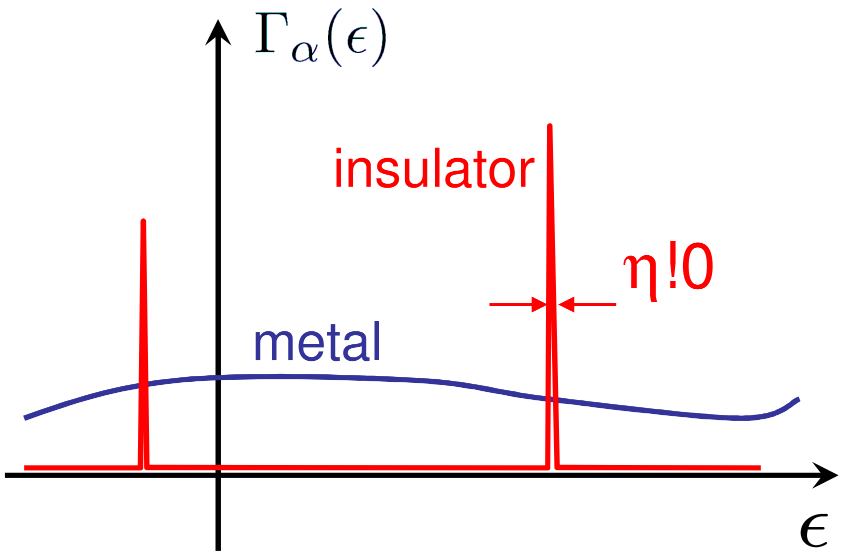
V.L. Berezinskii, (1973)

Many-body localization;

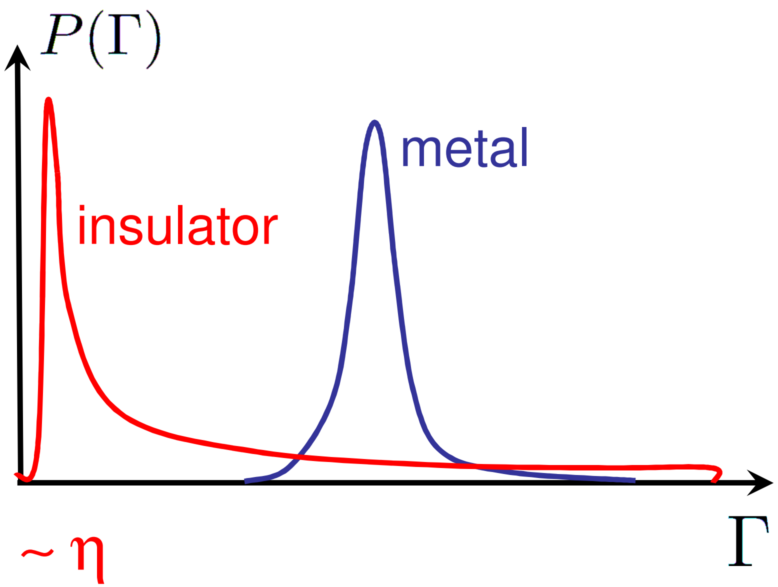
Idea for one particle localization Anderson, (1958);
MIT for Cayley tree: Abou-Chakra, Anderson, Thouless (1973);
Critical behavior: Efetov (1987)

$\Gamma_\alpha(\epsilon) = \mathrm{Im}\,\Sigma_\alpha^A(\epsilon)$ – random quantity

No interaction: $\Gamma_\alpha(\epsilon) = \eta \to +0$



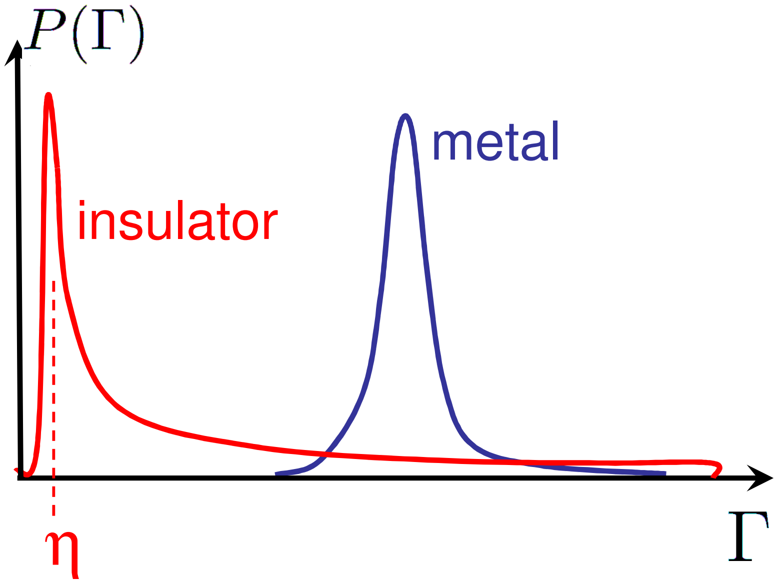
behavior for a
given realization



probability distribution
for a fixed energy

Perturbation theory for the fermionic systems:

$$\Gamma = \sum_{n,m} \Gamma^{n,m} \qquad P(\Gamma^{n,m}) = \sqrt{\frac{\gamma^{n,m}}{\pi\left[\Gamma^{n,m}\right]^3}} \exp\left(-\frac{\gamma^{n,m}}{\Gamma^{n,m}}\right)$$



probability distribution
for a fixed energy

$$\gamma^{n,m} \le \eta \left(\frac{T}{T_c}\right)^n$$

$$T_c = \frac{\delta_\zeta}{12\lambda M|\ln\lambda|}\left[1+\mathcal{O}\left(\lambda M \ln I\right)\right]$$

$T < T_c$ **STABLE**

$T > T_c$ ***unstable***

+ stability of the metallic phase at T À T_c

Estimate for the transition temperature for general case

1) Identify elementary (one particle) excitations and prove that they are localized.

2) Consider a one particle excitation at finite T and the possible paths of its decays:

$$¢_t(T_c)\ '\ \ U(T_c)N_1(T_c)$$

Energy mismatch

Interaction matrix element

# of possible decay processes of an excitations allowed by interaction Hamiltonian;

Fermionic system:

$$¢_t(T_c) \; ' \; U(T_c)N_1(T_c)$$

$$¢_t \; ' \; ±_3$$

$$U \; ' \; ¸ \, ±_3$$

$$N_1 \; ' \; \frac{T}{±_3}$$

# of electron-hole pairs

$$T_c \; ' \; \frac{±_3}{¸}$$

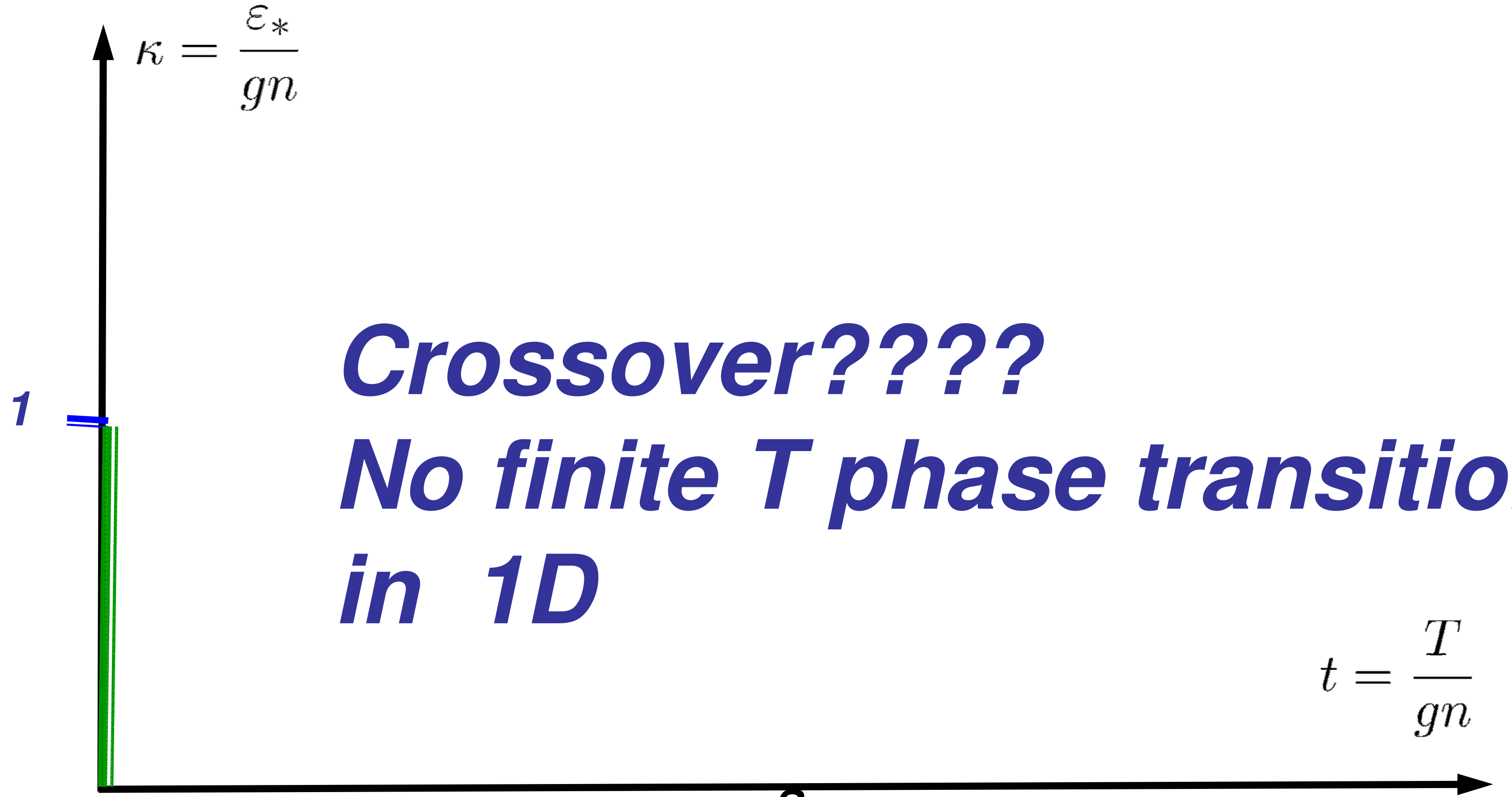
Weakly interacting bosons in one dimension

$$\hat{H} = \int_0^L dx \left[\hat{\psi}^\dagger \left(-\frac{\hbar^2 \partial_x^2}{2m} + V(x) \right) \hat{\psi} + \frac{g}{2} \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi} \right],$$

$$n = \frac{1}{L} \int_0^L dx \hat{\psi}^\dagger(x) \hat{\psi}(x)$$

$$0 < \gamma = \frac{gm}{n} \ll 1; \quad L \to \infty$$

Phase diagram



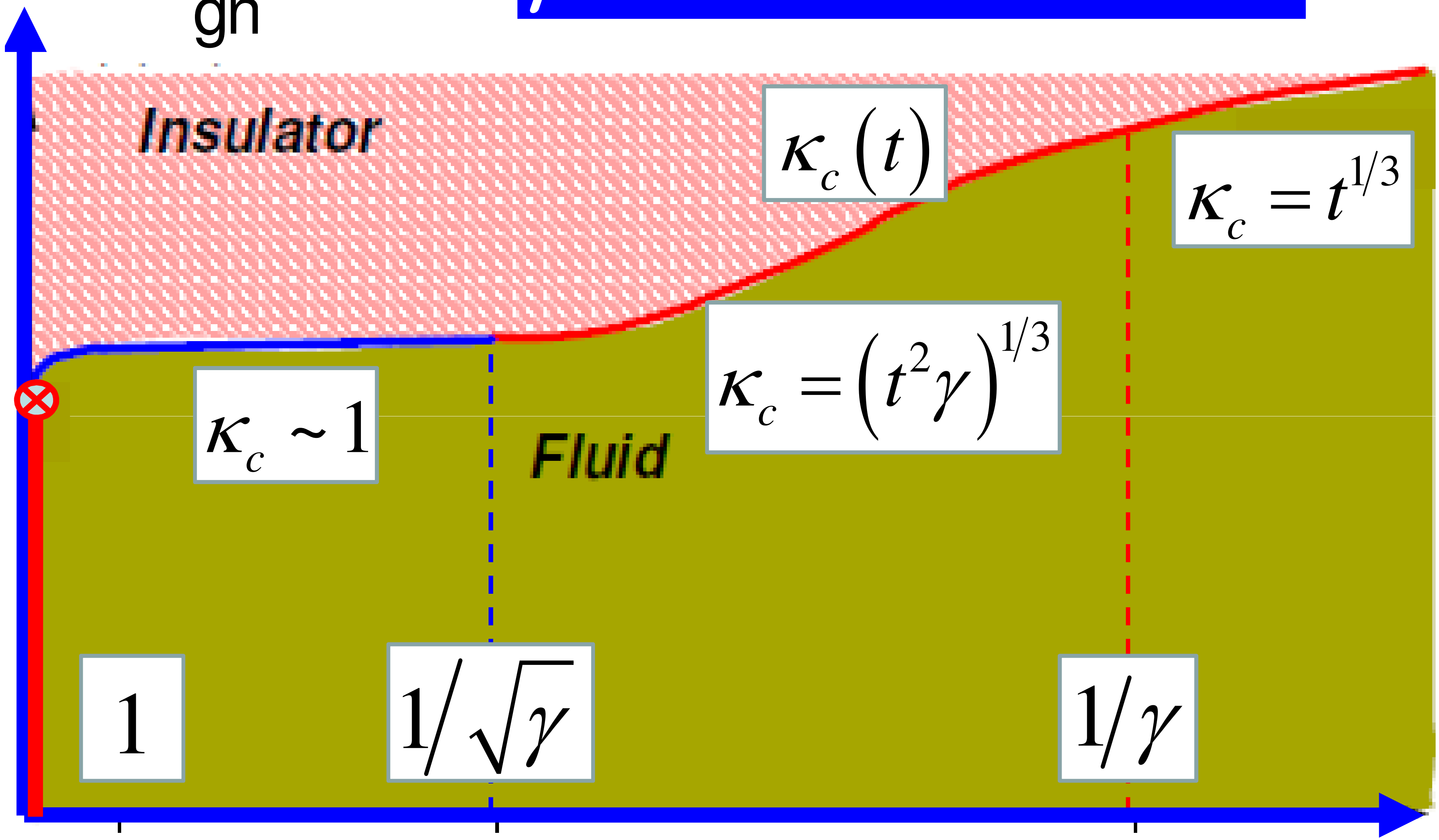
Crossover????

No finite T phase transition in 1D

$$T = 0 \quad \begin{array}{l} \kappa < 1; \quad \text{superfluid} \\ \kappa > 1; \quad \text{insulator} \end{array}$$

See e.g.
Altman, Kafri, Polkovnikov, G.Refael, PRL, 100, 170402 (2008); 93,150402 (2004).

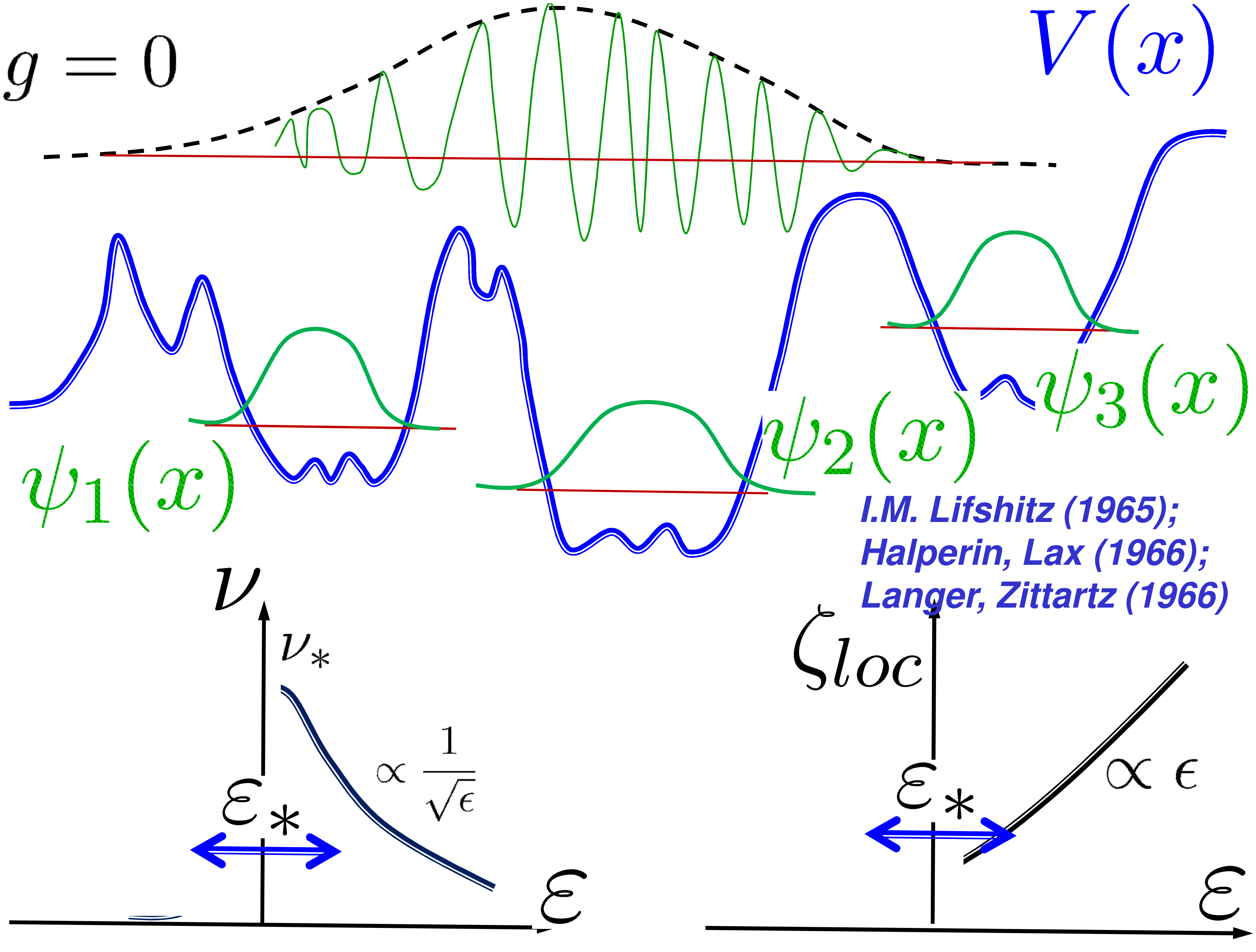
Finite temperature phase transition in 1D



$\gamma = \frac{gm}{n} \ll 1$

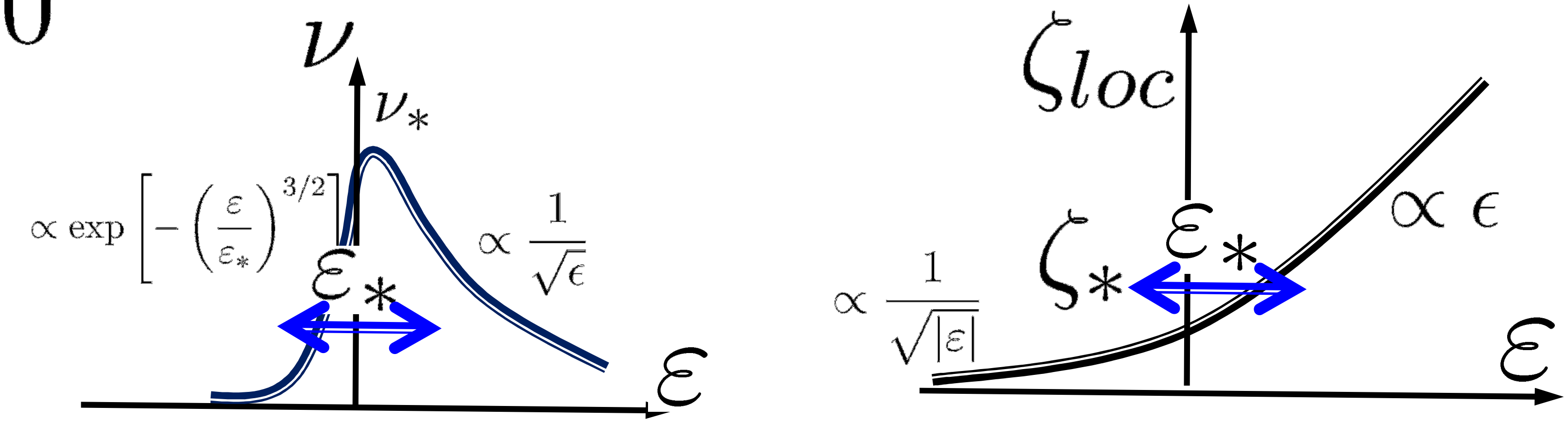
I.A., Altshuler, Shlyapnikov

arXiv:0910.434; Nature Physics (2010)



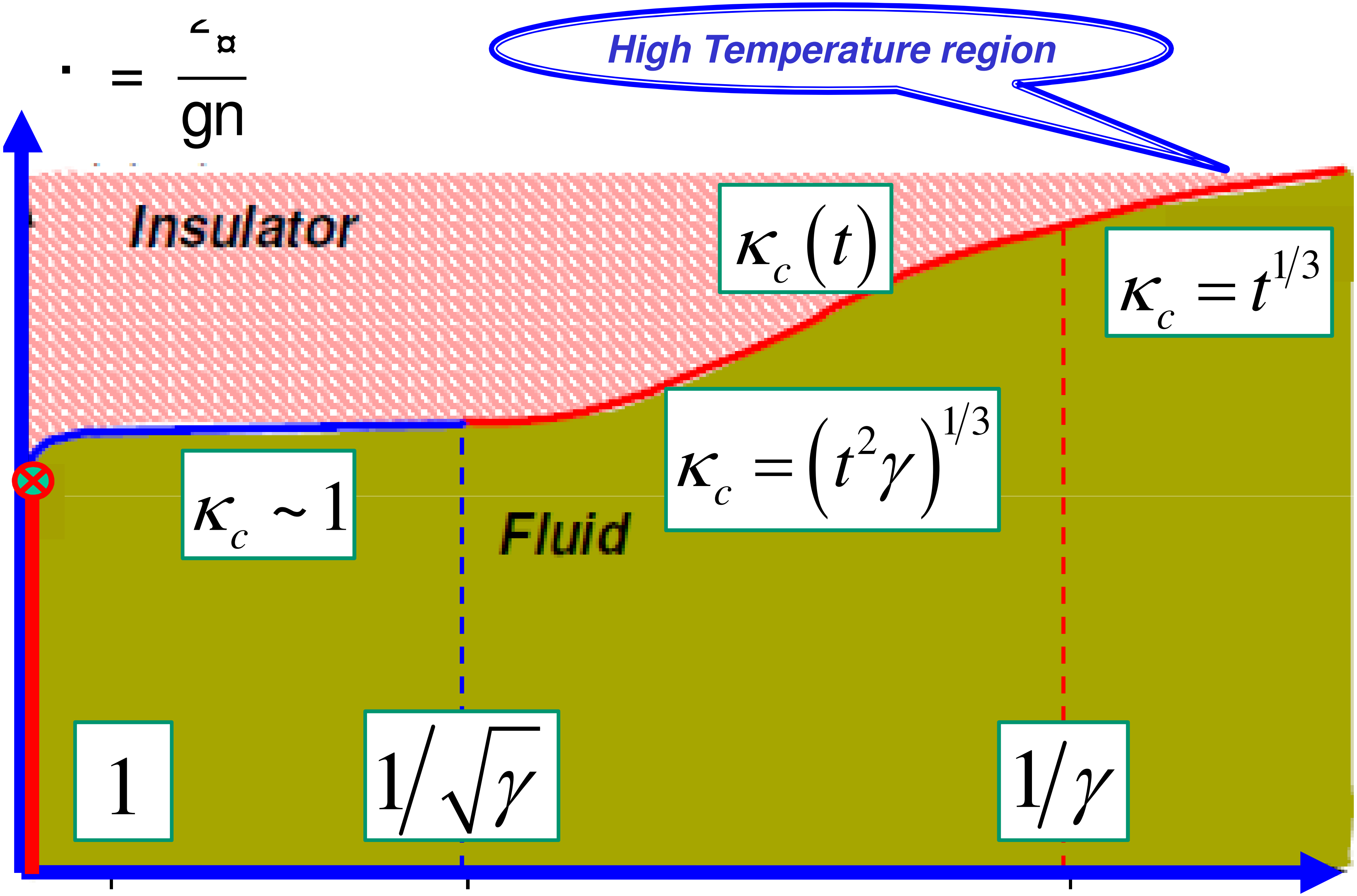
$g = 0$
$V(x)$
$\psi_1(x)$
$\psi_2(x)$
$\psi_3(x)$
I.M. Lifshitz (1965);
Halperin, Lax (1966);
Langer, Zittartz (1966)
ν
ν_*
$\propto \frac{1}{\sqrt{\epsilon}}$
ε_*
ε
ζ_{loc}
$\propto \epsilon$
ε_*
ε

$g = 0$



$$\varepsilon_* = \left[\left(\frac{m}{\hbar^2}\right)^2 \int dx \langle V(0)V(x)\rangle\right]^{1/3}$$

$$\zeta_* = \frac{\hbar}{\sqrt{m^2 \varepsilon_*}}$$



$t \equiv T/ng$

$\gamma = \frac{gm}{n} \ll 1$

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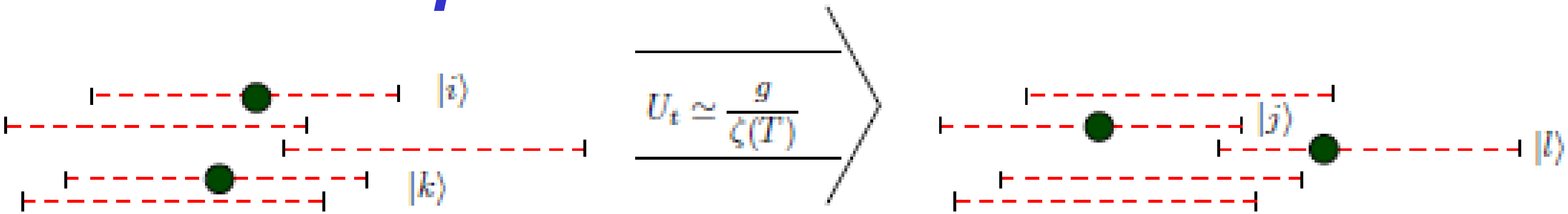
$t >> \gamma^{-1}$

Bose-gas is not degenerate: occupation numbers either 0 or 1



$$¢_t(T_c)\ '\ U(T_c)N_1(T_c)$$

$$¢_t\ '\ ±_3(T) = {}^2¤\ ˘\left(\frac{{}^2¤}{T}\right)^{1=2}$$

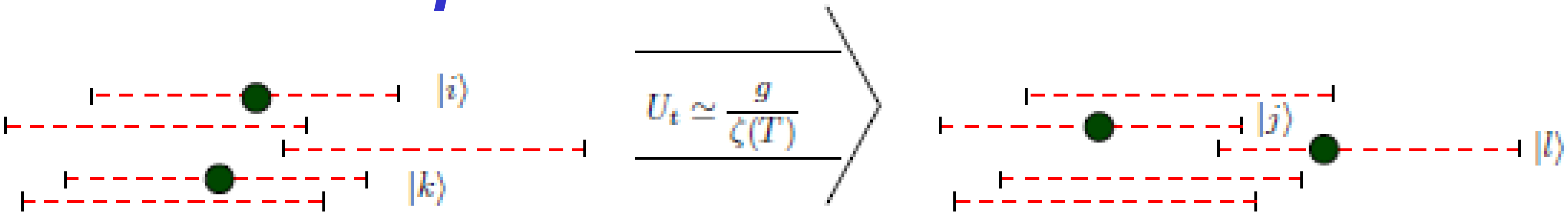
$$U\ '\ \frac{g}{{}^3(T)}$$

$$N_1\ '\ n^3(T)$$

# of bosons to interact with

$t >> \gamma^{-1}$

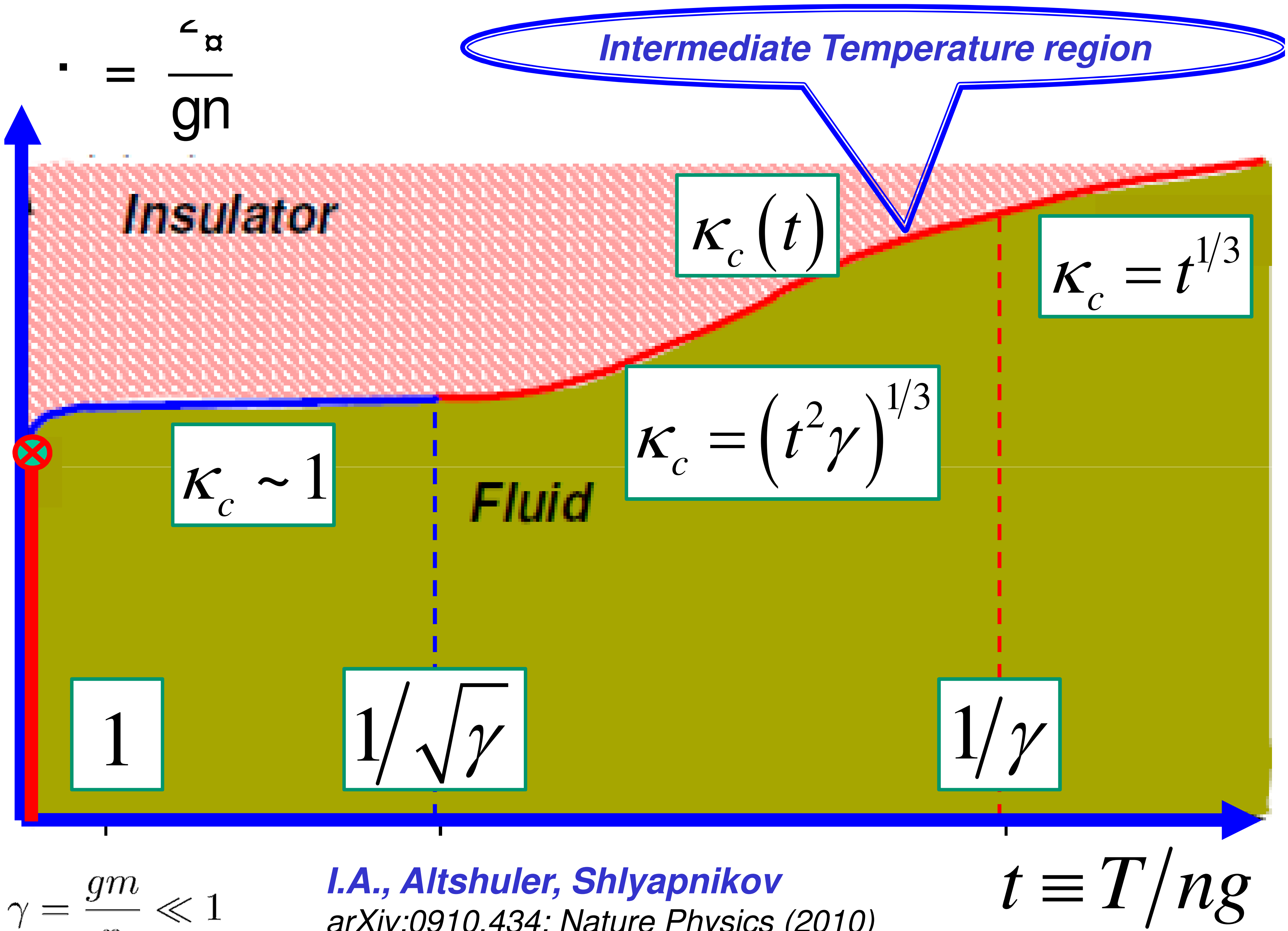
Bose-gas is not degenerate: occupation numbers either 0 or 1



$¢_t(T_c)$ ' $U(T_c)N_1(T_c)$

$2¤$ µ $\frac{2¤}{T_c}$ ¶$^{1=2}$ ' gn

▪ ' $t^{1=3}$



$\gamma = \frac{gm}{n} \ll 1$

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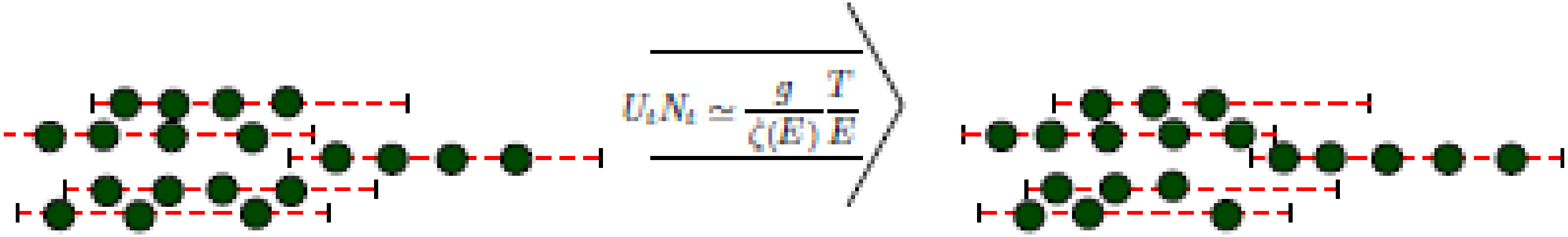
Intermediate temperatures: $\gamma^{-1/2} << t << \gamma^{-1}$

$$|\mu| = T^2 / T_d >> ng, E_* \qquad T << T_d$$

Bose-gas is degenerate; typical energies ~ $|\mu| << T$

occupation numbers $>>1$ matrix elements are enhanced



$$\kappa_c(t) \propto t^{2/3} \gamma^{1/3} \qquad \sqrt{\gamma} << t\gamma << 1$$

Intermediate temperatures:

$$\gamma^{-1/2} << t << \gamma^{-1}$$

Consider bosons with energies 2 [²; 2²], ² . T.

$$¢_t(T_c;{}^2) \text{ ' } U(T_c;{}^2)N_1(T_c;{}^2)$$

$$¢_t \text{ ' } ±_3({}^2) = {}^2{}_{¤} \left(\frac{{}^2{}_{¤}}{2} \right)^{1=2}$$

$$U \text{ ' } \frac{g}{3({}^2)} £ \left[\frac{T}{2} \right]$$

$$N_1 \text{ ' } \frac{{}^2}{±_3({}^2)}$$

# of levels with different energies

Intermediate temperatures:

$$\gamma^{-1/2} << t << \gamma^{-1}$$

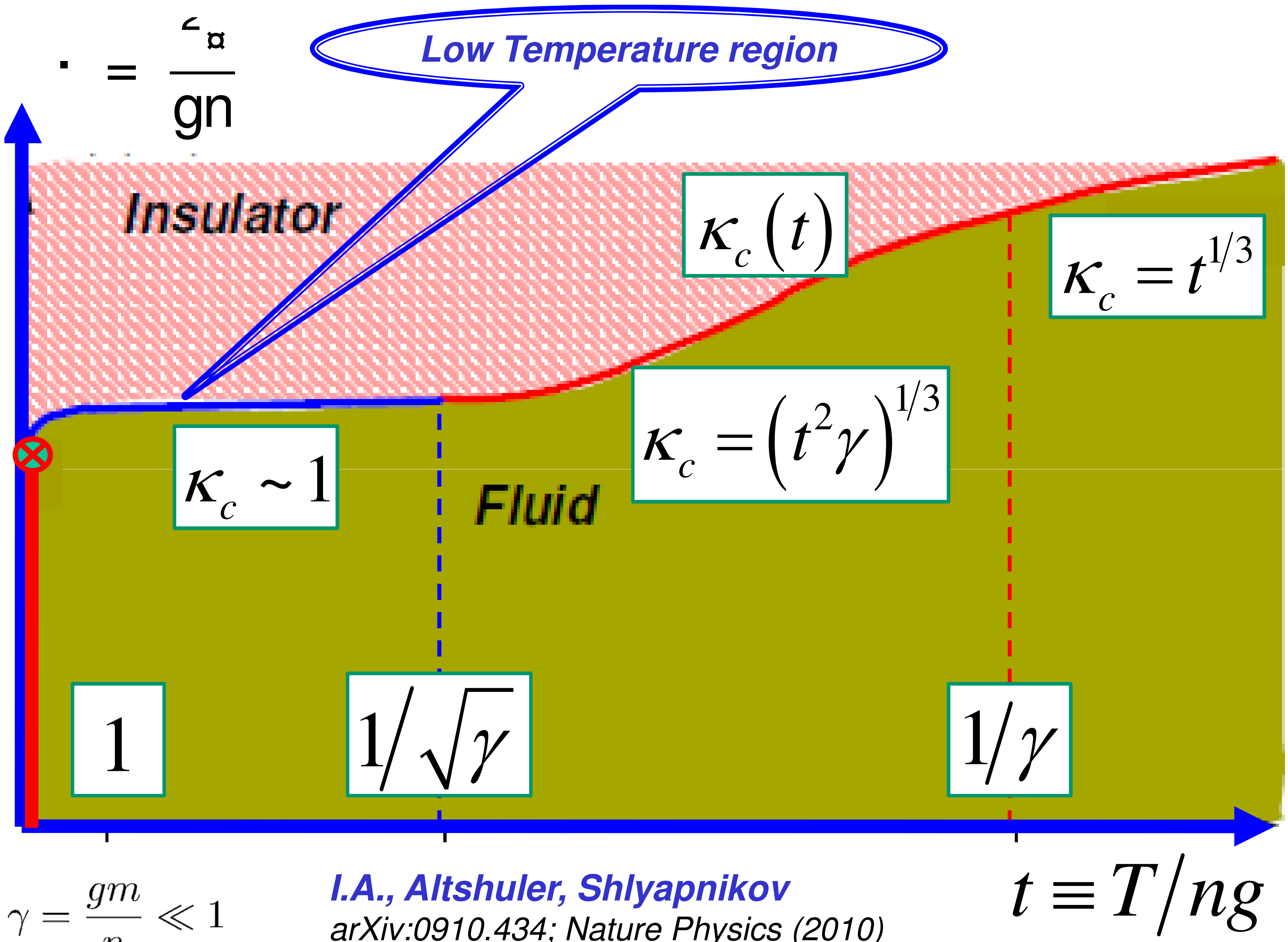
Consider bosons with energies 2 [²; 2²], ² . T.

¢ t (Tc; ²) ' U(Tc; ²)N1(Tc; ²)

T ' ³(²)±₃(²)² / g = ³¤ (~⁴ / m²³⁴¤) / g µ ² / ²¤ ¶ µ r (—— / 2¤ / 2) ¶ 2

Delocalization occurs in all energy strips

$$\kappa_c = \left(t^2\gamma\right)^{1/3}$$



$\gamma = \frac{gm}{n} \ll 1$

I.A., Altshuler, Shlyapnikov

arXiv:0910.434; Nature Physics (2010)

Low temperatures: $t << \gamma^{-1/2}$ **Start with *T=0***

Spectrum is determined by the interaction but only Lifhsitz tale is important;

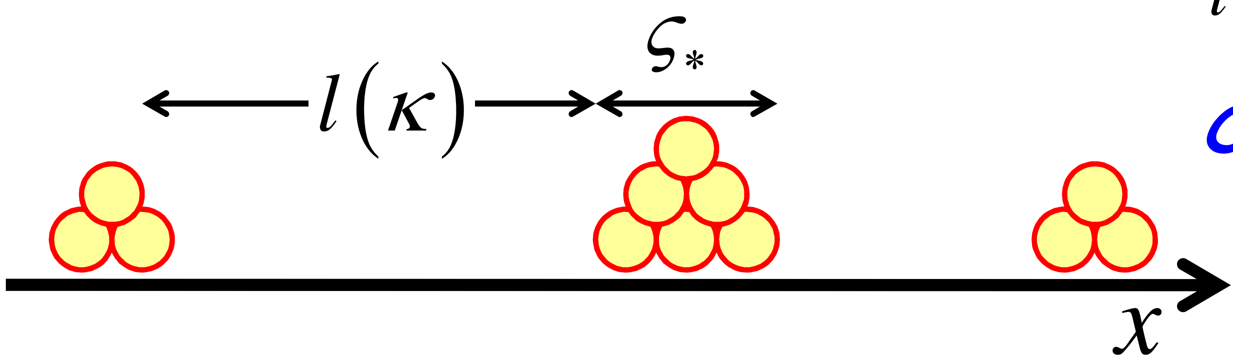
Gross-Pitaevskii mean-field on strongly localized states:

$$E(n_¡) = (^2¡ \; ¡ \; ^1)\, n_¡ + \frac{g}{2^3¤} n_¡^2$$

Optimal occupation # $$N_¡ = \frac{(^1 \; ¡ \; ^2¡)^3¤}{g}$$ ***Random, non-integer:***

$$\hat{H} = \sum_¡ \frac{g}{2^3¤} (\hat{n}_¡ \; ¡ \; N_¡)^2$$

$$l(\kappa) = \varsigma_* \sqrt{\kappa} >> \varsigma_*$$

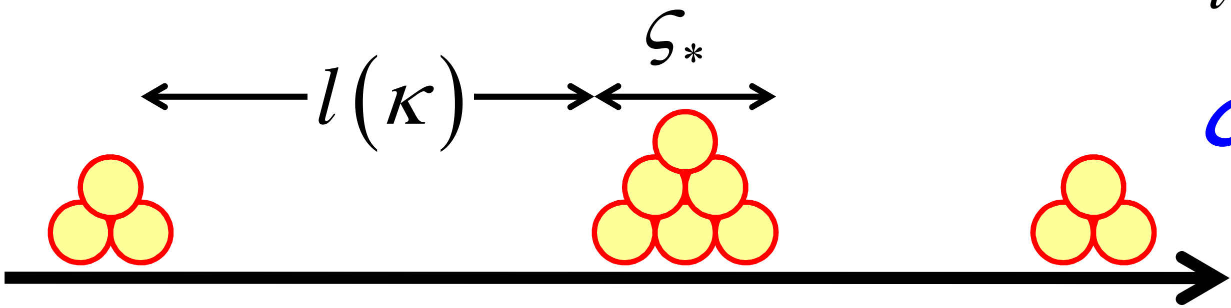


Occupation

$$n l(\kappa) / \varsigma_* = \gamma^{-1/2} >> 1$$

Low temperatures: $t << \gamma^{-1/2}$ **Start with *T=0***

$$l(\kappa) = \varsigma_* \sqrt{\kappa} >> \varsigma_*$$



Occupation

$$n l(\kappa)/\varsigma_* = \gamma^{-1/2} >> 1$$

$$\hat{H} = \sum_i \frac{g}{2\varsigma_*}(\hat{n}_i - N_i)^2$$

$$J_i \approx {}^{2}_{*}\sqrt{N_i N_{i+1}}\exp(-r_i/\varsigma_*)$$

$$p(J) \approx \frac{1}{J} J^{1/(2\sqrt{\gamma})}$$

$$p(r_i) \propto \exp[-r_i/(2\varsigma_*\sqrt{\gamma})]$$

See Altman, Kafri, Polkovnikov, G.Refael, PRL, 100, 170402 (2008); 93,150402 (2004).

Low temperatures: $t \ll \gamma^{-1/2}$ Start with *T=0*

$$\hat{H} = \sum_i \left[\frac{g}{2\gamma} (\hat{n}_i - N_i)^2 + J_i \cos\left(\hat{\phi}_i - \hat{\phi}_{i+1}\right) \right]$$

$$p(J) \simeq \frac{1}{J} J^{1/(2\sqrt{\gamma})}$$

Everything is determined by the weakest links:

T=0 transition:

$J < J_0;\quad J_0 \to 0$ $\qquad$ $J < J_0;\quad J_0 \to 0$

$$L \propto \left(\frac{1}{J_0}\right)^{\frac{1}{\sqrt{\gamma}}}$$

Insulator

Interaction relevant: $g/L > J_0$ if $\gamma > 1$

Interaction irrelevant: $g/L < J_0$ if $\gamma < 1$

"Superfluid"

Low temperatures: $t \ll \gamma^{-1/2}$ **Start with *T=0***

$$\hat H = \sum_i \frac{g}{2\gamma}(\hat n_i - N_i)^2 + J_i \cos(\hat\phi_i - \hat\phi_{i+1})$$

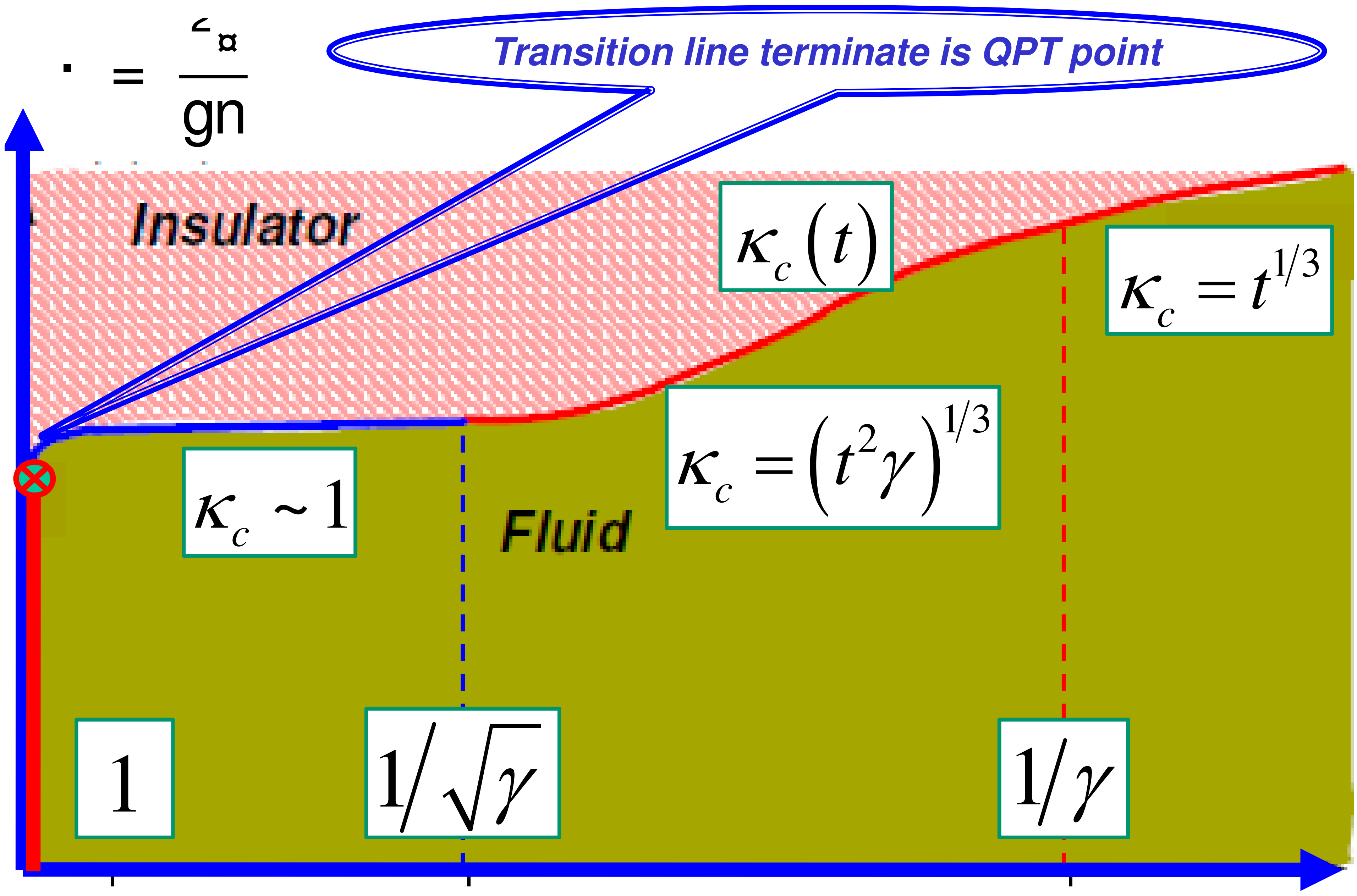
$$p(J) \approx \frac{1}{J} J^{1/(2\sqrt{\gamma})}$$

Insulator:

All excitations are localized; many-body Localization transtion temperature finite;

"Superfluid"

Localization length of the low-energy excitations (phonons) diverges As their energy goes to zero; The system is delocalized at any finite Temperature;



$\gamma = \frac{gm}{n} \ll 1$

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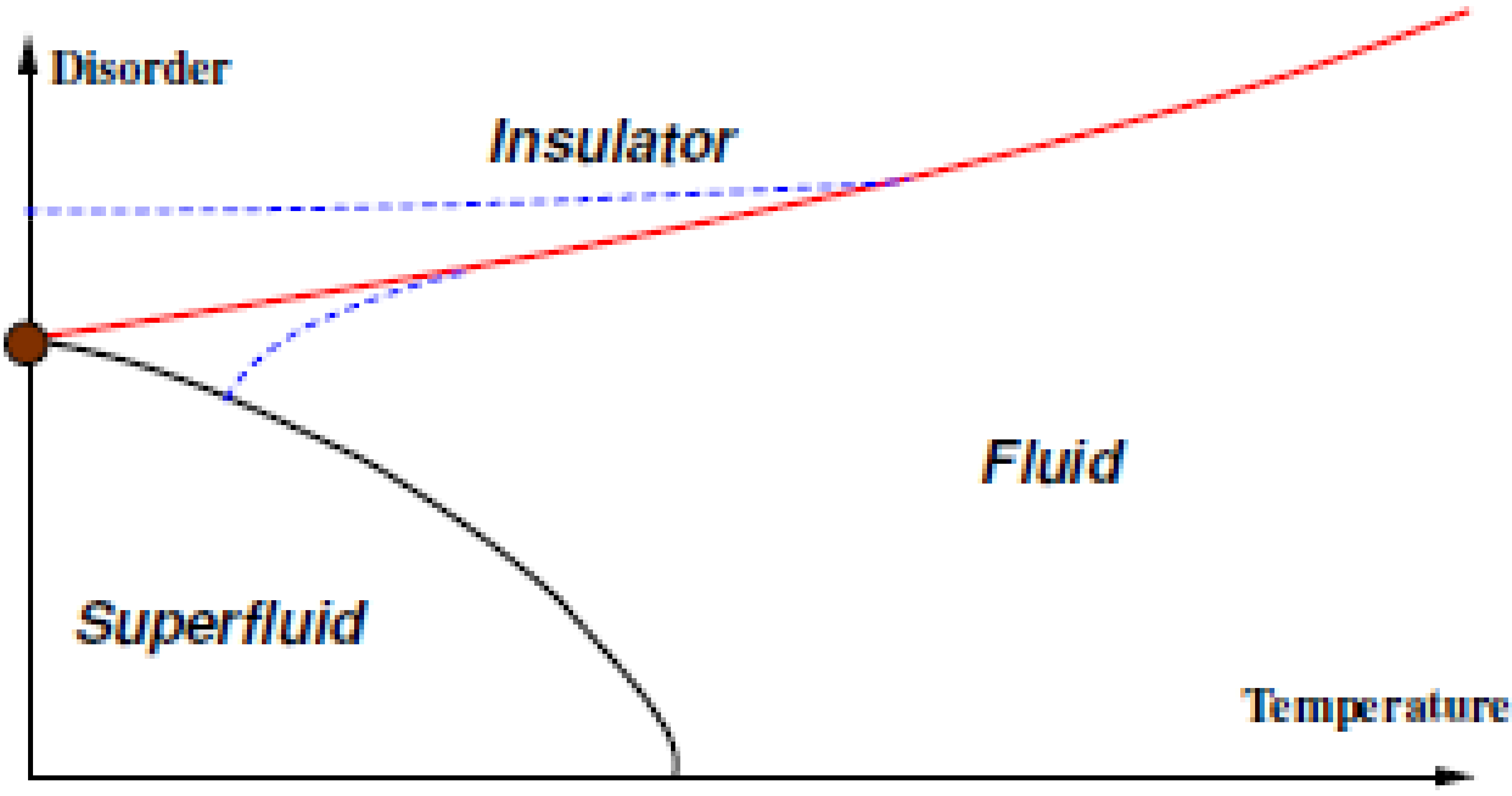
$t \equiv T/ng$

Disordered interacting bosons in two dimensions (conjecture)



Conclusions:

- Existence of the many-body mobility threshold is established.
- The many body metal-insulator transition is ***not*** a thermodynamic phase transition.
- It is associated with the vanishing of the Langevine forces rather the divergences in energy landscape (like in classical glass)
- Only phase transition possible in one dimension (for local Hamiltonians)