Dynamics of Thin Solitons

Lev Pitaevskii
INO-CNR BEC Center, University of Trento;
Kapitza Institute for Physical Problems
Moscow.

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The goal of the work

Let we know equation of state

$\mu = \mu(n)$ of the medium and

the energy of a flat soliton $E_S(V,\mu)$

in a uniform medium. Can we solve

more complicated problems?

A standard example. GP-equation for the condensate wave function

$$i\hbar\partial_t\Psi = -\frac{\hbar^2}{2m}\partial_x^2\Psi + g\Psi\left|\Psi\right|^2 + U\left(x\right)\Psi$$

$$g = \frac{4\pi\hbar^2}{m}a > 0, \quad a \text{ is the scattering length}$$

$$\Psi = \left|\Psi\right|e^{i\Phi}, n\left(x,t\right) = \left|\Psi\left(x,t\right)\right|^2, v = \frac{\hbar}{m}\partial_x\Phi$$

“Grey” soliton in an uniform condensate

$$\Psi\left(x - X(t)\right) = \sqrt{\bar{n}}\left[i\frac{V}{c} + \frac{u}{c}\tanh\left(\frac{mu}{\hbar}\left(x - X(t)\right)\right)\right]$$

$$X(t) = Vt\,, \quad u = \sqrt{c^2 - V^2}\,, \quad c = \sqrt{\frac{g\bar{n}}{m}}\,, \quad \frac{n_{\min}}{\bar{n}} = \frac{V^2}{c^2}$$

$$\Delta\Phi = 2\arccos\left(\frac{V}{c}\right)$$

T. Tsuzuki (1971)

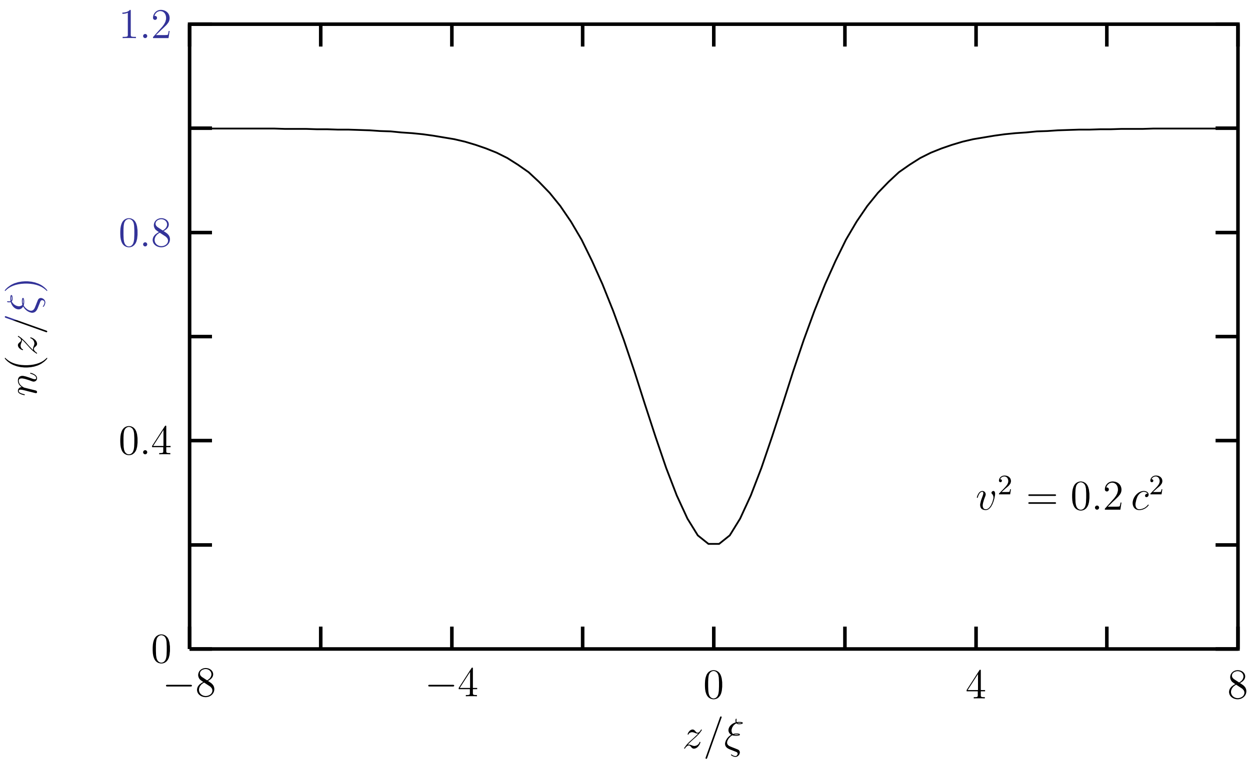
Density perturbation

$$\delta n(x) = -\bar{n}\frac{u^2}{c^2}\frac{1}{\mathrm{ch}^2\left[\frac{mu}{\hbar}(x-X)\right]}$$

$$X = Vt, u = \sqrt{c^2 - V^2}$$

$$\text{Soliton length}: \xi = \hbar / mu$$

Density profile



Parameters of a BEC soliton

$$E_s = \frac{4\hbar m}{3g}\left(c^2 - V^2\right)^{3/2} \equiv \frac{4\hbar m}{3g} u^3$$

$$c^2 = \frac{\mu}{m}, N_s = -\frac{\partial E_s}{\partial \mu} = -\frac{2\hbar}{g} u$$

$$u \equiv \left(c^2 - V^2\right)^{1/2}$$

General system

Order parameter

$\Psi = |\Psi| e^{i\Phi}$ still exists.

However current $j \neq |\Psi|^2 \hbar \partial_x \Phi$

Non-local relation between

j and $\partial_x \Phi$.

Fermionic superfluid!

Condensate in external potential. Local density approximation (LDA)

$$L_X >> \xi$$

$$\mu \to \mu - U(X)$$

Energy conservation

$$E_s(V, \mu - U(X)) = const$$

Dynamics of soliton in LDA. Energy conservation.

$$\frac{dE_S}{dt} = \frac{dE_S}{dV^2} 2V \frac{dV}{dt} - \frac{dE_S}{d\mu}\frac{dU}{dX} V = 0$$

$$\text{"Newton equation"}: m_I \frac{dV}{dt} = -N_S \frac{dU}{dX}$$

$$\text{"Inertial mass"}: m_I \equiv 2\frac{dE_S}{dV^2} < 0$$

Number atoms in soliton

$$N_S \equiv \int_{-\infty}^{\infty} [n(x) - \overline{n}]dx = -\frac{dE_S}{d\mu} < 0$$

BEC example

$$\text{In BEC:} \quad E_S = E_S\left(\mu / m - V^2\right)$$

$$N_S = \frac{m_I}{2m}, m^* \equiv \frac{m_I}{N_S} = 2m$$

Soliton moves as a paticle of mass $2m$

V. Konotop, L. Pitaevskii, 2004

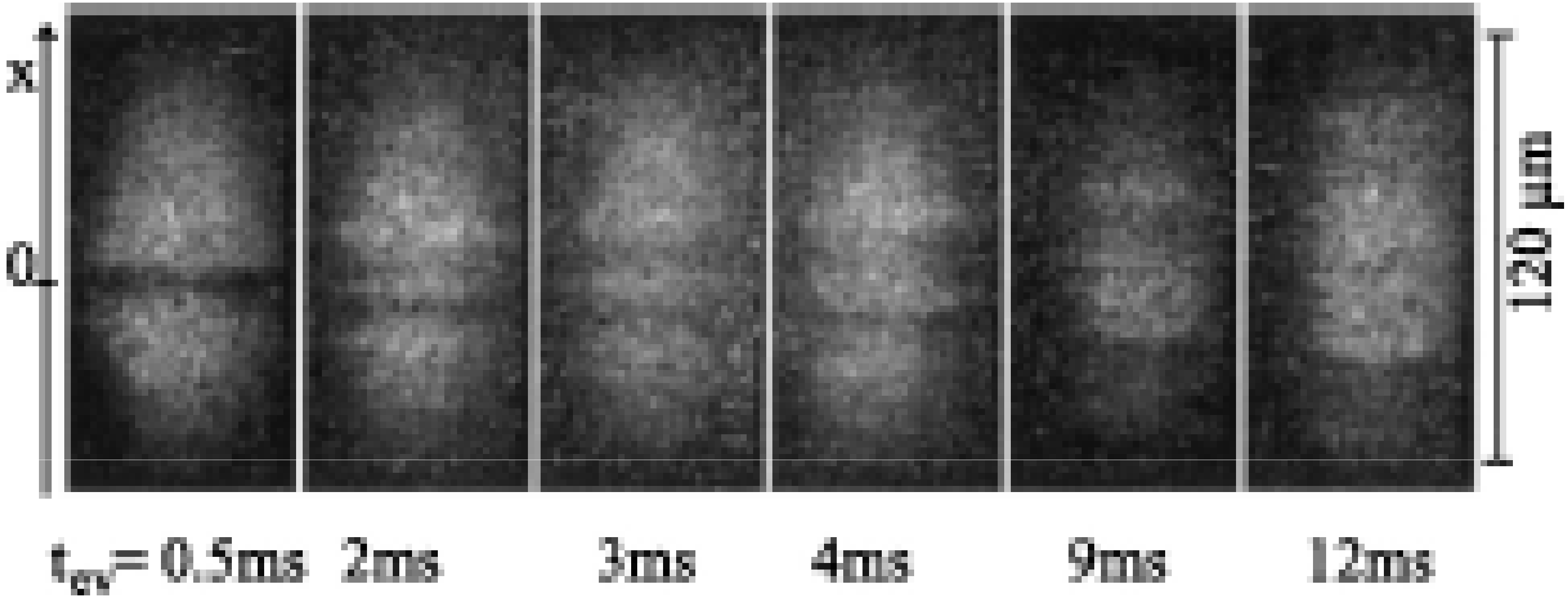
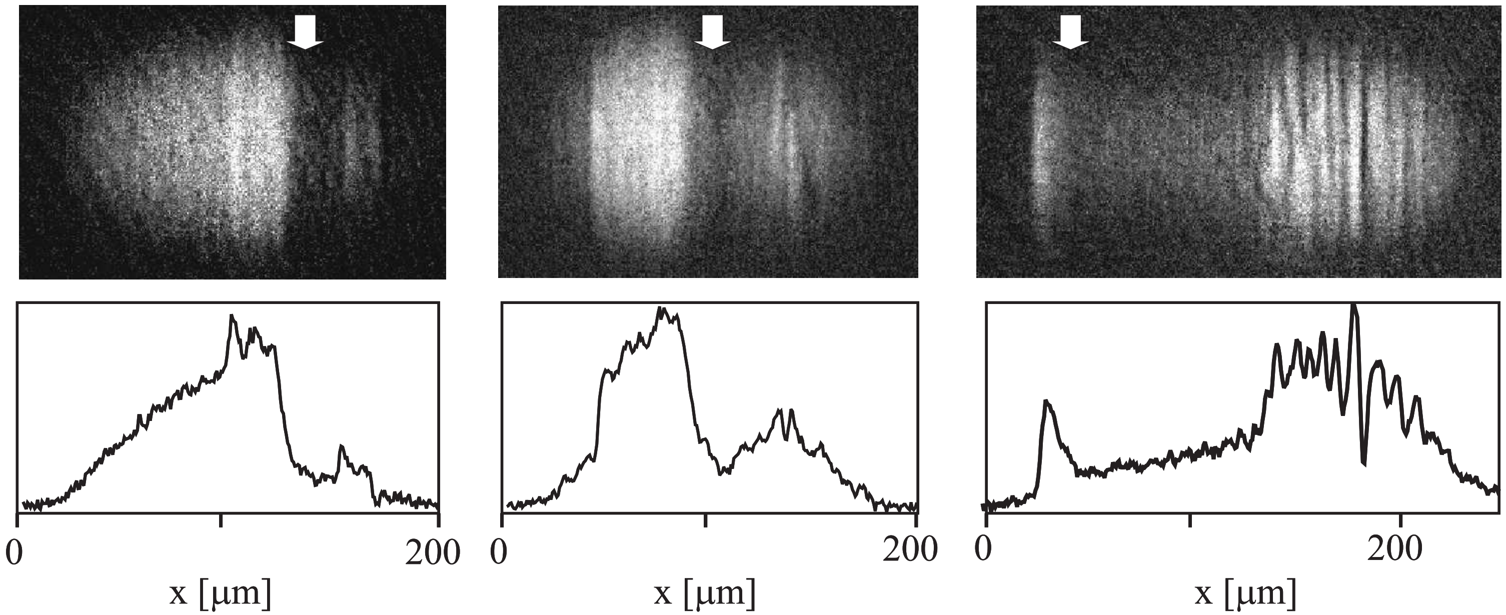


FIG. 2. Absorption images of BEC's with kink-wise structures propagating in the direction of the long condensate axis, for different evolution times in the magnetic trap, t_{ev}. ($\Delta\phi \sim \pi$, $N \simeq 1.5 \times 10^5$, and $t_{TOF} = 4$ ms).

Burger et al., 1999

Solitons in BEC



Engels and Atherton, 2007

Harmonic trap

$$U(x) = m\omega_x^2 x^2 / 2 \,,$$

Frequency of small oscillations

of a soliton :

$$\omega = \omega_x \left(\sqrt{\frac{mN_S}{m_I}} \right)_{V \to 0}$$

BEC in a harmonic trap

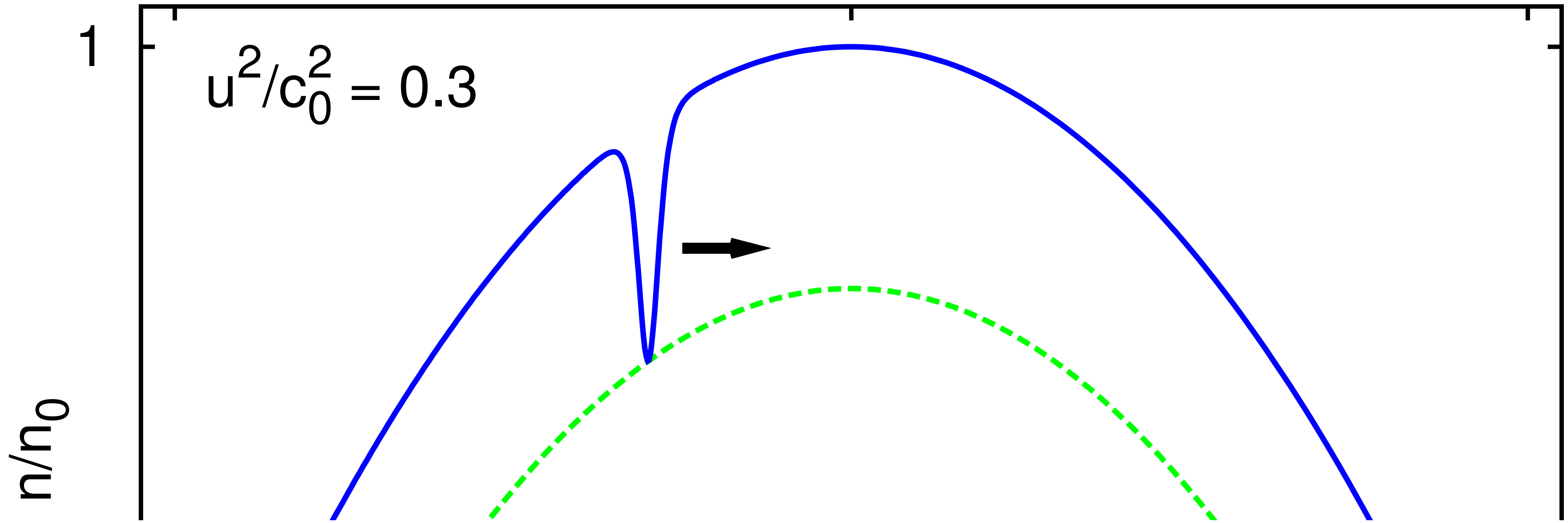
For BEC in a trap:

$$\omega = \omega_x / \sqrt{2}.$$

For small oscillations:

T. Bush, J. Anglin, 2000.

Actually is valid for arbitrary amplitudes.



1
$u^2/c_0^2 = 0.3$
n/n_0

ERROR: invalidrestore
OFFENDING COMMAND: restore

STACK:

-savelevel-
-savelevel-
-dictionary-