Quantum memories:
design and applications

Condensed Matter Seminar,
TECHNION, December 4th, 2012

MPQ
Max-Planck-Institut
für Quantenoptik

QUANTUM MEMORIES

$$|\Psi\rangle = a|0\rangle + b|1\rangle \qquad\qquad |\Psi'\rangle \approx |\Psi\rangle$$

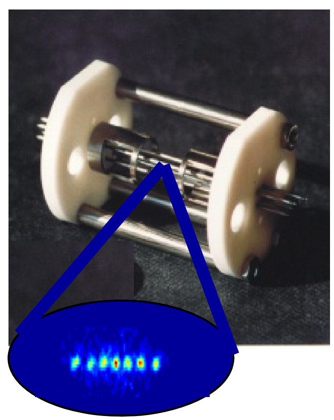
- **Decoherence:** external noise, coupling to environment, etc
- **Goal:** memory time as long as possible

- **Applications**:
  - Cryptography
  - Quantum repeaters
  - Quantum money

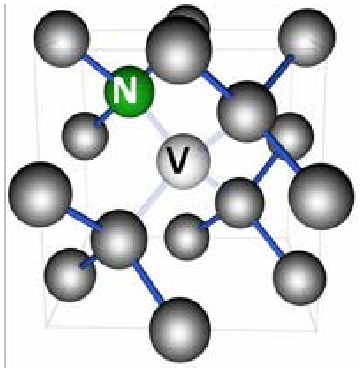
QUANTUM MEMORIES
APPROACHES

1. ISOLATION + DECOUPLING

TRAPPED IONS



NV-CENTERS



- Memory times of the order of hours
- Require complex set-ups

- Memory times of the order seconds
- Relative simple set-ups

2. FAULT-TOLERANT QUANTUM ERROR CORRECTION

- Quantum computation: Identity operator

- General local (few particle) errors:
  - Hamiltonians (eg, errors in gates)
  - Interaction with environment (eg, depolarizing noise)

- Error threshold:

$$P_{\text{error/step}} \leq P_{\text{threshold}} \sim 10^{-4}$$

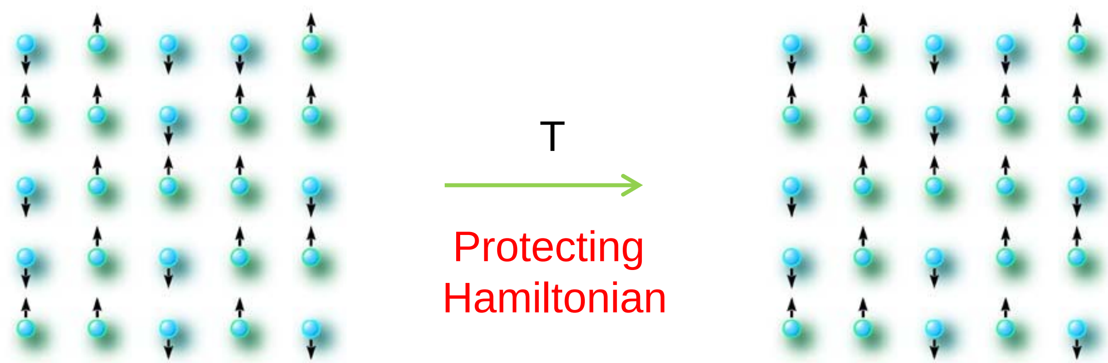
- Memory time:

$$T \sim e^{kN}$$

3. SELF-PROTECTING QUANTUM MEMORIES

- Original idea: Kitaev
- Like classical memories (eg Hard disk):

$$|\Psi\rangle = a|0\rangle + b|1\rangle \qquad \rho \sim |\Psi\rangle\langle\Psi|$$



SELF-PROTECTING QUANTUM MEMORIES

- Which errors can tolerate?
  - Hamiltonian perturbations (local)
  - Interaction with environment (eg, depolarizing noise)

- Memory time? $T \sim f(N)$

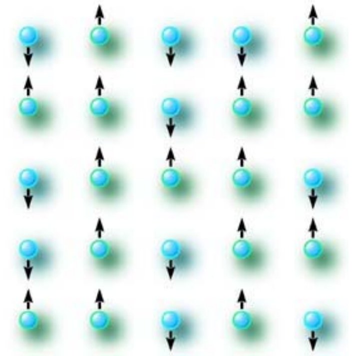
OUTLINE

1. Depolarizing noise
2. Hamiltonian perturbations
3. Applications
4. Quantum simulations of HEP

1. Depolarizing noise

F. Pastawski, A. Kay, N. Schuch, JIC,Phys Rev. Lett.**103**, 080501 (2009)
F..Pastawski, L.Clemente, JIC, Phys.Rev. A **83**, 012304 (2011)

1. DEPOLARIZING NOISE



$$|\Psi\rangle = a|0\rangle + b|1\rangle$$

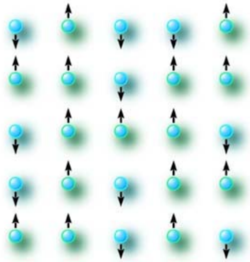
- Markovian Depolarizing Noise:

$$\dot{\rho} = -i[H_{\text{protecting}}, \rho] + \Gamma \sum_n L_n(\rho)$$

$$L_n(\rho) = \frac{1}{2}\mathrm{tr}_n(\rho) - \rho$$

depolarization rate

1. DEPOLARIZING NOISE



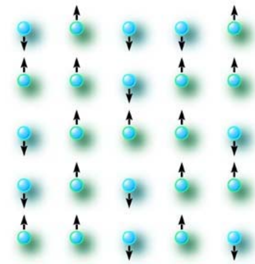
1.1. NO PROTECTING HAMILTONIAN

$$\rho(t) = \mathrm{E}_t^{\otimes N}(\rho)$$

- After $T = \ln 3 / \Gamma$, E_T is an entanglement breaking channel
- Quantum information cannot withstand such channel

➡ The memory time is independent of N

1. DEPOLARIZING NOISE



1.2. PROTECTING HAMILTONIAN

$$\dot{\rho} = -i[H_{\text{protecting}}, \rho] + \Gamma \sum_n L_n(\rho)$$

- For ANY Hamiltonian, $\frac{dI(t)}{dt} \leq -\Gamma I(t)$

$I = N - S(\rho)$

(information content)

- After a time $T = \ln(2N)/\Gamma$, the information content $I \leq 1/2$
- If $I \leq 1/2$, no information can be stored

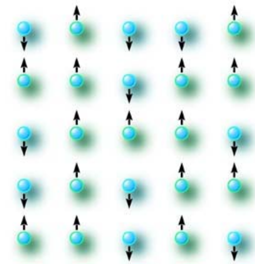
➡ The memory time is at most $\sim \log(N)$

the bound can be reached!

1. DEPOLARIZING NOISE

1.3. CONCLUSIONS

$$\dot{\rho} = -i[H_{\text{protecting}}, \rho] + \Gamma \sum_n L_n(\rho)$$



- A protecting Hamiltonian helps.
- The time only scales logarithmically

- We need to get rid of entropy ➡ dissipation
- For other noises, it may be better

1. DEPOLARIZING NOISE

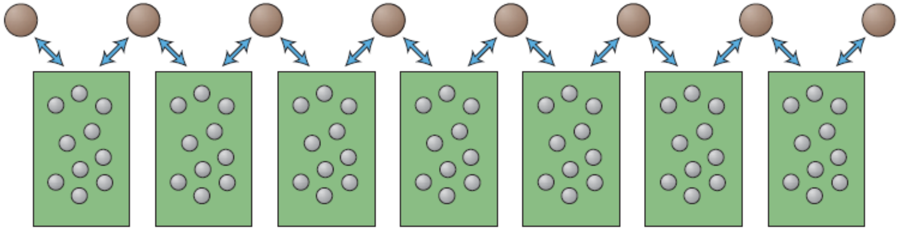
1.4. DISSIPATIVE PROTECTION

- Idea: replace protecting Hamiltonian by protecting dissipation

$$\dot{\rho} = -i[H_{\text{protecting}}, \rho] + \Gamma \sum_n L_n(\rho)$$

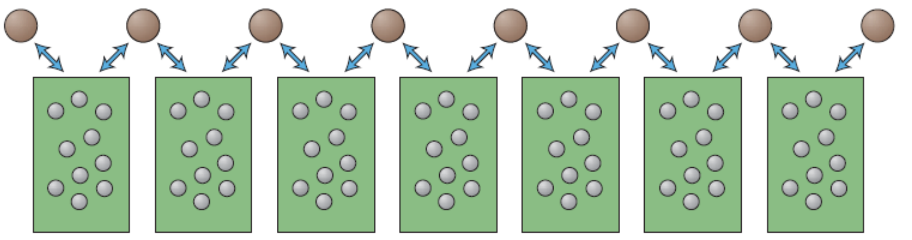
$$\Downarrow$$

$$\dot{\rho} = L^{\text{protecting}}(\rho) + \Gamma \sum_n L_n(\rho)$$



1. DEPOLARIZING NOISE

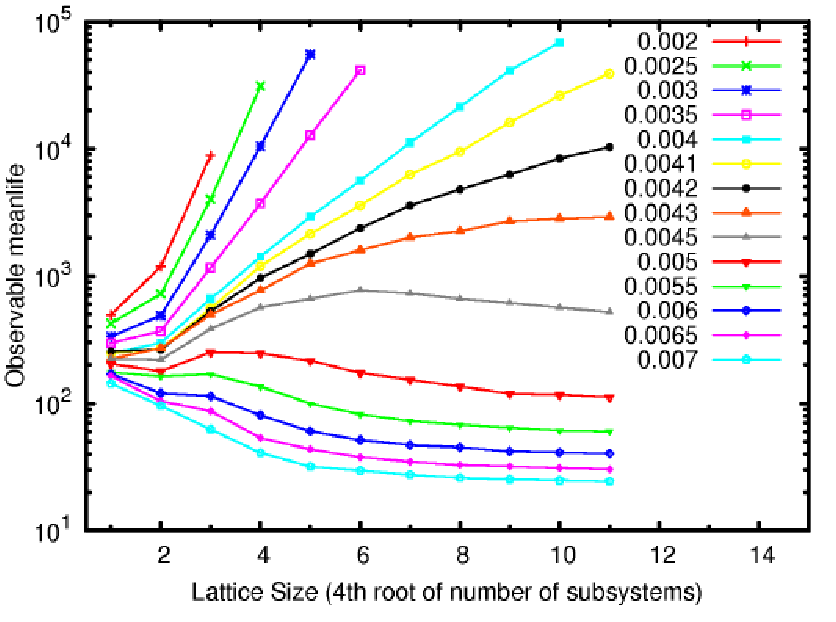
1.4. DISSIPATIVE PROTECTION



$$\dot{\rho} = L^{\text{protecting}}(\rho) + \Gamma \sum_n L_n(\rho)$$

- Gets rid of entropy
- Corrects all local errors
- Can the dissipation be local?

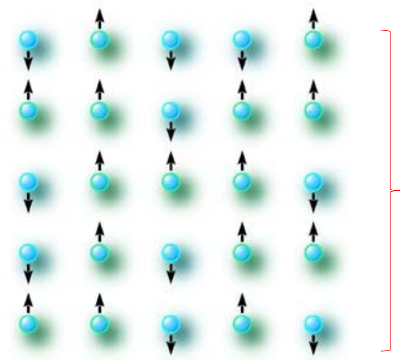
For the moment, in 4D (local)



2. Hamiltonian noise

F. Pastawski, A. Kay, N. Schuch, JIC, QIC**10,** 0580-0618 (2010)
L. Mazza, M. Rizzi, M. Lukin, JIC (in preparation)

2. HAMILTONIAN NOISE



$H_{\text{protecting}}$

$$|\Psi\rangle = a|0\rangle + b|1\rangle$$

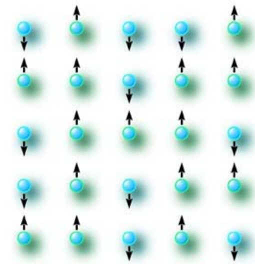
- Small perturbation

$$|\dot{\Psi}\rangle = -i(H_{\text{protecting}} + \varepsilon V)|\Psi\rangle$$

local perturbation $V = \sum_n V_n$

Is the qubit protected for all local perturbations?

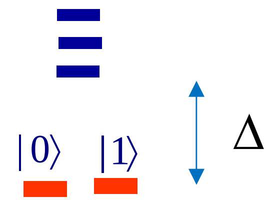
2. HAMILTONIAN NOISE



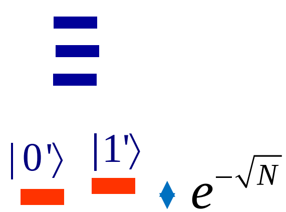
2.1. MAIN IDEA

- H has a degenerate ground state with a gap

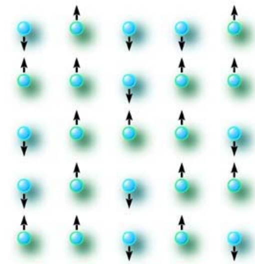
$$|\Psi_0\rangle \rightarrow |0\rangle$$
$$|\Psi_1\rangle \rightarrow |1\rangle$$



- Local perturbations mildly lift the degeneracy



➡ No accumulation of phase-errors



2.2. ADVERSARY HAMILTONIAN

- Find a particular perturbation

$$|\dot{\Psi}\rangle = -i(H_{\text{protecting}} + \varepsilon V)|\Psi\rangle$$

- Prove that for that particular one, the information cannot be recovered

➡ It cannot protect against ALL local perturbations

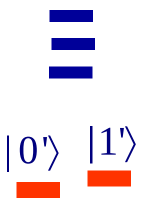
- Simple perturbation:

$$H = U^{\dagger} H_{\text{protecting}} U \quad \leftarrow \quad U = e^{-i\varepsilon \sum_n h_n}$$

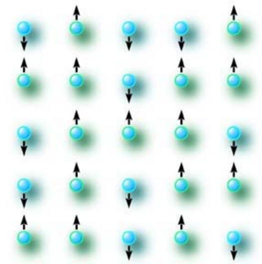
$$\varepsilon V = U^{\dagger} H_{\text{protecting}} U - H_{\text{protecting}} = \varepsilon \sum_n V_n$$

2. HAMILTONIAN NOISE

2.3. ANDERSON CATASTROPHY



$|0'\rangle \quad |1'\rangle$ ♦ $e^{-\sqrt{N}}$



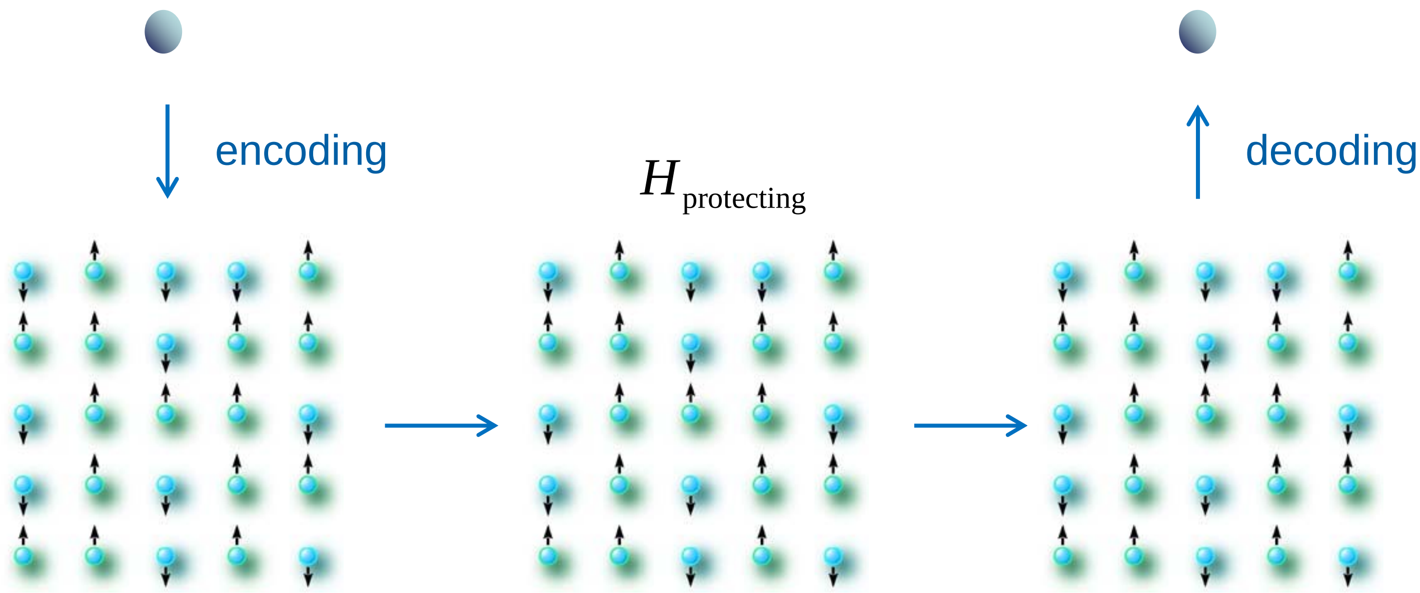
$$|\langle 0|0'\rangle| \leq e^{-N}$$

➡ A decoding operation (error correction) at the end is required

2. HAMILTONIAN NOISE

$$|\Psi\rangle = a|0\rangle + b|1\rangle$$

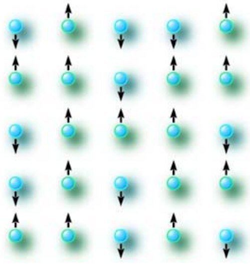
$$\rho \sim \Psi$$



$$\rho = \mathrm{E}_2 \bullet \mathrm{D}_T \bullet \mathrm{E}_1(\Psi)$$

➡ Use QECC

2. HAMILTONIAN NOISE



2.4. MODELS:

- Kitaev‘s toric code (2D):
  Bacon‘s compass model (3D):

$$T \sim \log(N)$$

- Including randomness: $T \sim N$

- Including time-dependent perturbations: $T \sim 1$

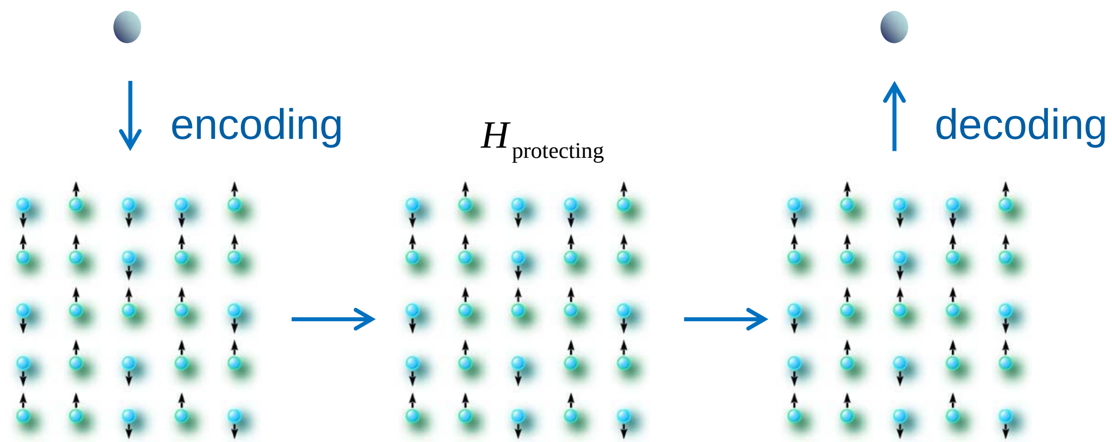
See also

E. Dennis, A. Kitaev, A. Landahl, and J. Preskill, Jour. Math. Phys. 43, 4452 (2002).
R. Alicki, M. Horodecki, P. Horodecki, and R. Horodecki, arXiv:0811.0033 (2008).
S. Chesi, D. Loss, S. Bravyi, and B. M. Terhal, arXiv:0907.2807 (2009).
S. Bravyi, D. P. DiVincenzo, D. Loss, and B. M. Terhal, Phys. Rev. Lett. 101, 070503 (2008).

2. HAMILTONIAN NOISE

2.5. BEYOND QECC:

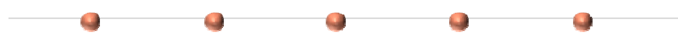
$|\Psi\rangle = a|0\rangle + b|1\rangle$ $\rho \sim \Psi$



$$\rho = \mathrm{E}_2 \bullet \mathrm{D}_T \bullet \mathrm{E}_1(\Psi)$$

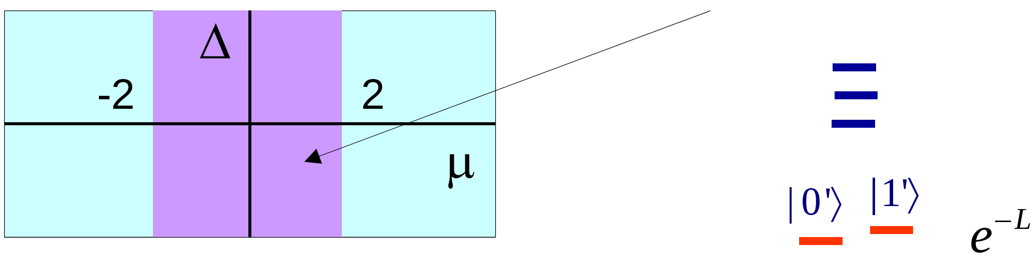
- Not necessarily an ECC
- We should find the optimal decoding operation

2.6. RESTRICTED ERRORS: MAJORANA FERMIONS



$$H = -\mu \sum_n a_n^\dagger a_n - \sum_n a_n^\dagger a_{n+1} + \Delta \sum_n a_n a_{n+1} + h.c.$$

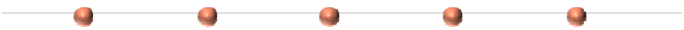
Phase diagram (T=0) Spectrum



- Hamiltonian perturbations conserve parity (SSR)
- Problem is Gaussian: Can be solved

2. HAMILTONIAN NOISE

2.6. RESTRICTED ERRORS: MAJORANA FERMIONS



$$H = -\mu \sum_n a_n^\dagger a_n - \sum_n a_n^\dagger a_{n+1} + \Delta \sum_n a_n a_{n+1} + h.c.$$

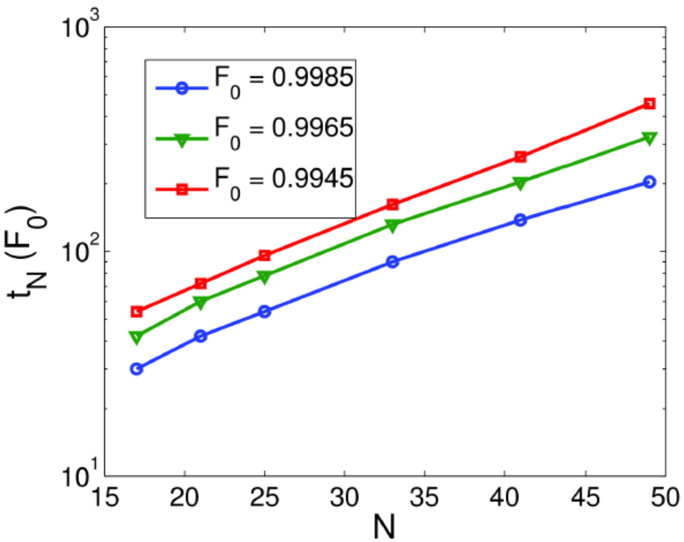
$$\varepsilon V = \sum_n \varepsilon_n(t) a_n^\dagger a_n + \ldots$$

- Average with respect to different noise realizations

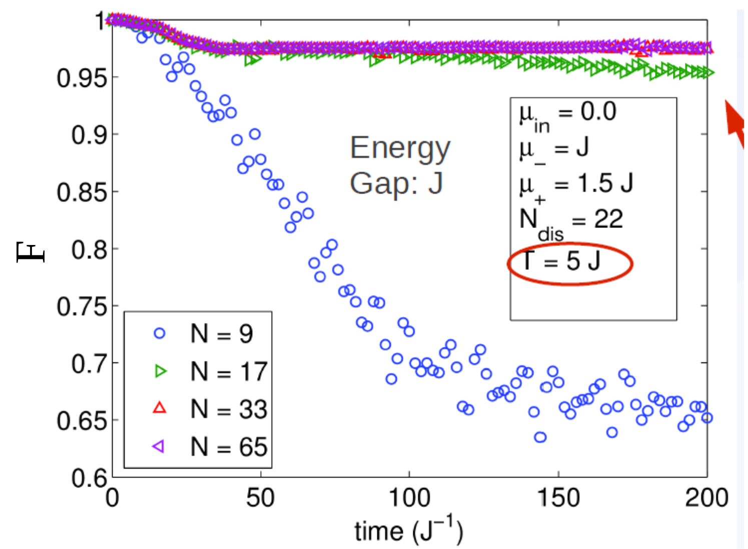
2. HAMILTONIAN NOISE

2.6. RESTRICTED ERRORS: MAJORANA FERMIONS

Scaling



Effects of temperature



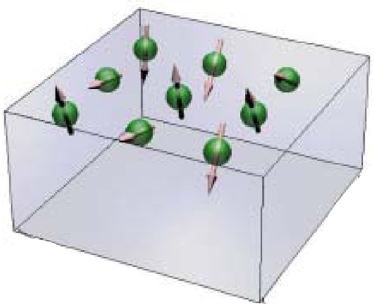
- Memory time is compatible with an exponential scaling
- At finite temperature, the time saturates

3. Applications

F. Pastawski, N. Yao, L. Yang, M.D. Lukin, JIC, arXiv:1112.5456

3. APPLICATIONS

- NV Centers:



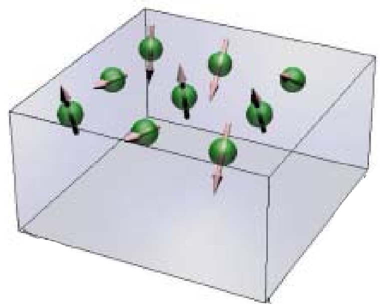
- Room temperature
- No vacuum, etc
- Magnetic shielding
- Many qubits

- Product sate:

$$|\alpha\rangle|\beta\rangle\ldots$$

3. APPLICATIONS

- NV Centers:



- Room temperature
- No vacuum, etc
- Magnetic shielding
- Many qubits

- Product sate:

$$|\alpha\rangle|\beta\rangle\ldots$$

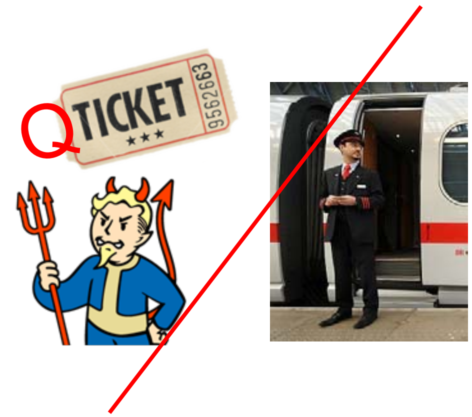
- Quantum money

Protocols: Wiesner 1969 (1983),
Mosca et al, 2007, with QC
Gavinsky 2011, with CC

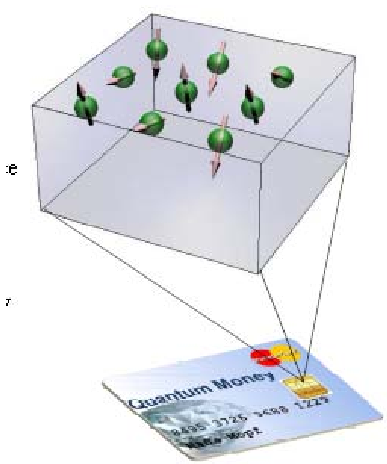
NO SECURITY PROOF SO FAR

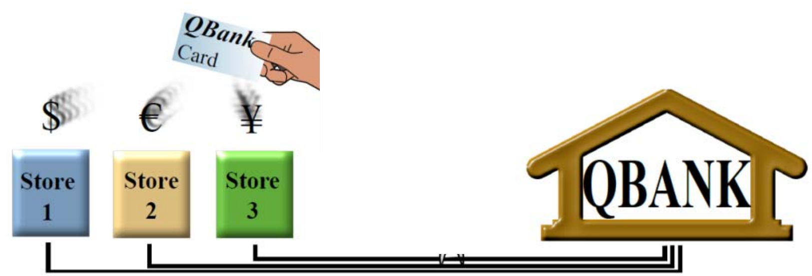
3. APPLICATIONS

- Quantum tickets:



- Classically verifiable tickets:





3. APPLICATIONS

3.1. SECURITY:

- Under realistic conditions, not all the qubits will give the correct outcome

  ➡ Some errors must be tolerated

- If the tolerance is too high, one could have made many copies

  ➡ $F > F_{\mathrm{tol}}$

- The user may learn by trying to verify his ticket many times

QUESTIONS:

- What is the minimum tolerance, such that the protocols are secure?
- How many times can a ticket be verified?

3.2. Q-TICKETS: Protocol

- Each q-ticket has a: - Classical serial number
  - N qubits, in a product state, randomly chosen

A quantum bank note, containing a secret set of polarized photons, cannot be copied by counterfeiters, who would disturb the photons by attempting to measure them.

$$|0\rangle, |1\rangle, |+\rangle, |-\rangle, |+i\rangle, |-i\rangle$$

- The verifies measures a random subset of qubits with:

  - acceptance $F > F_{\mathrm{tol}}$
  - no acceptance $F \leq F_{\mathrm{tol}}$

3. APPLICATIONS

3.2. Q-TICKETS: Security

- Soundness: honest owners can enter the train.

$$P_{\text{accept}}(F) \geq 1 - e^{-ND(F\|F_{tol})}$$

- Safety: no double success.

$$P_{\text{2 are accepted}} \leq e^{-ND(2F_{\text{tol}}-1\|2/3)} \quad \Rightarrow \quad F_{\text{tol}} > 5/6$$

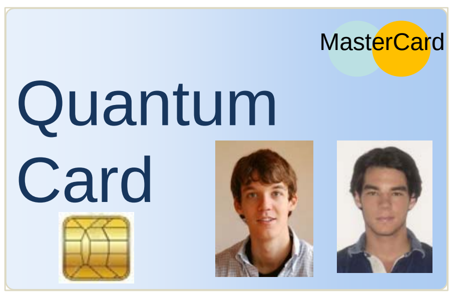
- Multipe verifications: $P_{\text{2 are accepted}}(v) \leq \binom{v}{2} e^{-ND(2F_{\text{tol}}-1\|2/3)}$

- Proof:

  - Assume general forging TPCP map.
  - Transform discrete problem to continuous via 3-designs.
  - Extend results on perfect cloning.
  - Chernoff bounds for non iid sources.

3.3. cv-TICKETS: Protocol

- Each q-ticket has a: - Classical serial number
  - N pairs of qubits, in a product state, randomly chosen



$$|0+\rangle, |0-\rangle, |1+\rangle, |1-\rangle, |+0\rangle, |-0\rangle, |+1\rangle, |-1\rangle$$

- Verification takes place remotely, with classical communication.
- Verifier asks random questions (XX or ZZ) which are non-informative.

Gavinsky, D. (2011). Quantum Money with Classical Verification. arXiv:1109.0372.

3.3. cv-TICKETS: Security

- Soundness: honest owners can pass the test.
- Safety: no double success, no simultaneous verification with many verifiers.

- Proof:

  - Same as before.
  - Extension of quantum retrieval games (Gavinsky)
  - Chernoff/Hoeffding and Impagliazzo/Kabanets bounds.

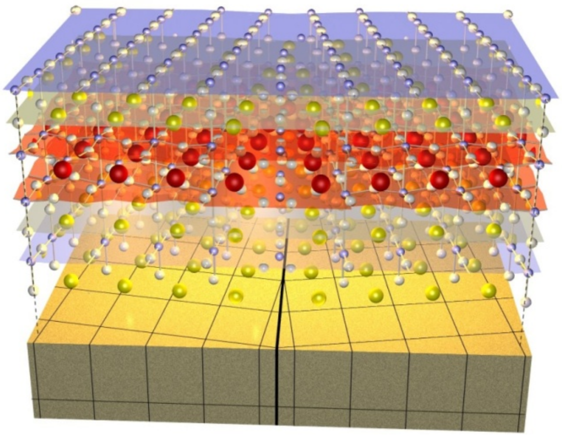
4. Quantum simulations

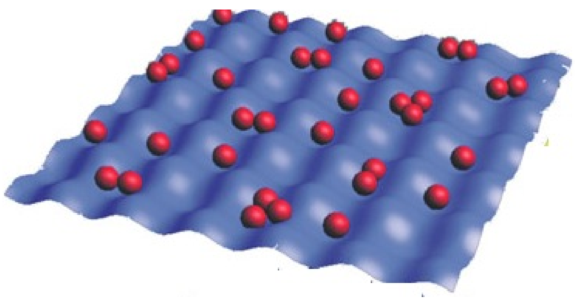
Zohar, IC, Reznik, PRL 109, 125302 (2012)
Zohar, IC, Reznik, arXiv:1208.4299

QUANTUM SIMULATION

PHYSICAL SYSTEM

QUANTUM SIMULATOR





Phenomenological Hamiltonian

$$H = \dots$$

Physical Hamiltonian

$$H = \dots$$

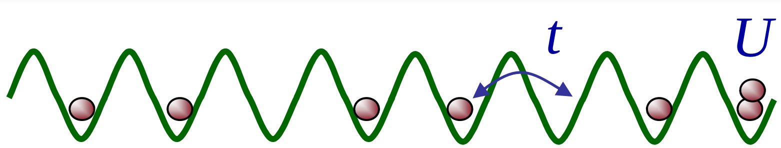
QUANTUM SIMULATION
ATOMS IN OPTICAL LATTICES

- Cold atoms are described by simple field theories:

$$H = \int \Psi_\sigma^\dagger \left(-\nabla^2 + V(r)\right)\Psi_\sigma + u_{\sigma_i} \int \Psi_{\sigma_1}^\dagger \Psi_{\sigma_2}^\dagger \Psi_{\sigma_3} \Psi_{\sigma_4}$$

  - One can also use external laser fields

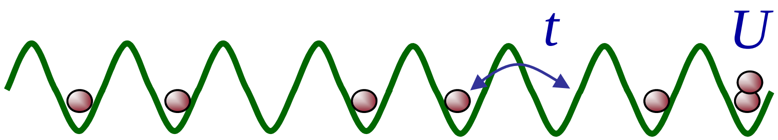
- Atoms in optical lattices: Low energies (temperatures):



Bose-Hubbard model

$$H = -t\sum_n \left(a_n^\dagger a_{n+1} + h.c\right) + U\sum_n a_n^{\dagger 2} a_n^2$$

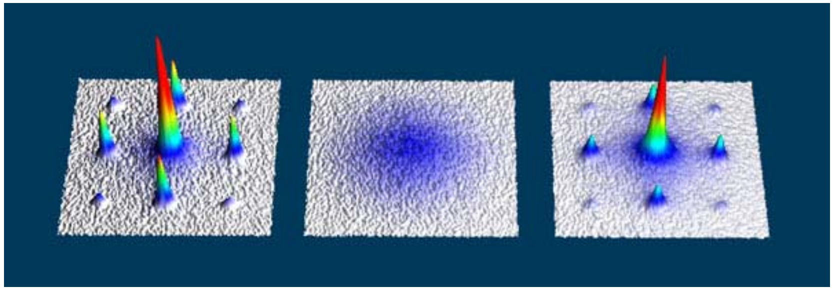
- Hubbard model: Mott insulator – superfluid tansition



Bose-Hubbard model

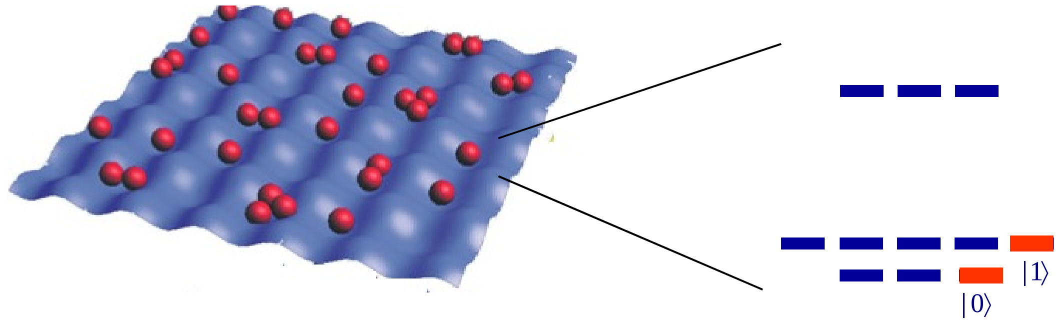
$$H = -t\sum_{n}\left(a_n^{\dagger}a_{n+1} + h.c\right) + U\sum_{n} a_n^{\dagger 2}a_n^{2}$$

- Experimentally observed



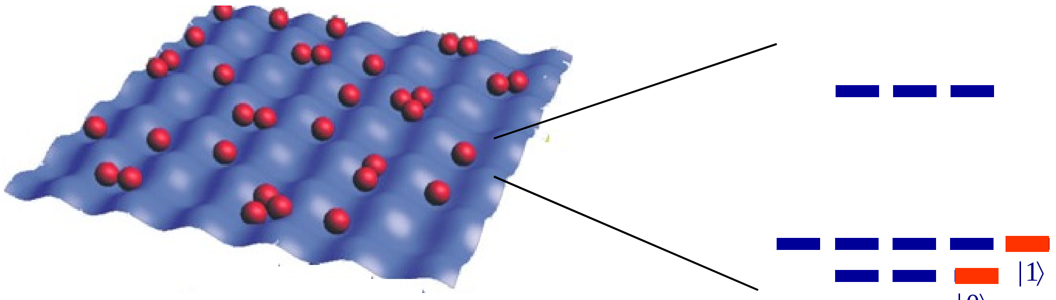
QUANTUM SIMULATION

ATOMS IN OPTICAL LATTICES



- Bosons/Fermions: $$H = -\sum_{\substack{<n,m> \\ \sigma,\sigma'}} \left(t_{\sigma,\sigma'} a^\dagger_{n,\sigma} a_{m,\sigma'} + h.c \right) + \sum_{\substack{n \\ \sigma,\sigma'}} U_{\sigma,\sigma'} a^\dagger_{n,\sigma} a^\dagger_{n,\sigma'} a_{n,\sigma'} a_{n,\sigma}$$
- Spins: $$H = -\sum_{\substack{<n,m> \\ \sigma,\sigma'}} \left(J_x S^x_n S^x_m + J_y S^y_n S^y_m + J_z S^z_n S^z_m \right) + \sum_{\substack{n \\ \sigma,\sigma'}} B_n S^z_n$$

CONDENSED MATTER PHYSICS



HIGH ENERGY PHYSICS?

- Fermions + Gauge Fields

$$L = \int \overline{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi - eQ\int A_{\mu}\overline{\Psi}\gamma^{\mu}\Psi - \frac{1}{4}\int F_{\mu\nu}F^{\mu\nu} + \ldots\ldots$$

➡ We need bosonic and fermionic atoms

We need interactions among themselves

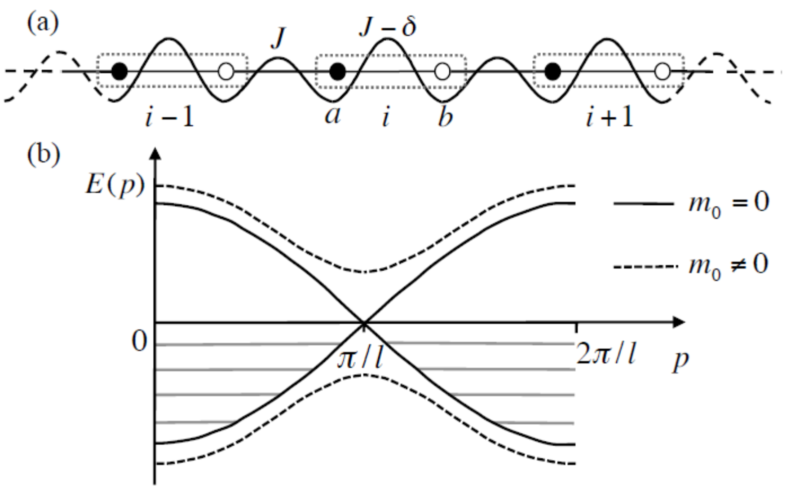
$$H = \int \Psi_{\sigma}^{\dagger}\left(-\nabla^{2} + V(r)\right)\Psi_{\sigma} + u\int \Phi_{\sigma}^{\dagger}\Phi_{\sigma'}\Psi_{\sigma}^{\dagger}\Psi_{\sigma'} + v\int \Phi_{\sigma}^{\dagger}\Phi_{\sigma'}^{\dagger}\Phi_{\sigma'}\Phi_{\sigma} + w\int \Psi_{\sigma}^{\dagger}\Psi_{\sigma'}^{\dagger}\Psi_{\sigma'}\Psi_{\sigma} + \ldots\ldots$$

- Relativistic

$$L = \int \overline{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi - eQ\int A_{\mu}\overline{\Psi}\gamma^{\mu}\Psi - \frac{1}{4}\int F_{\mu\nu}F^{\mu\nu} + \ldots\ldots$$

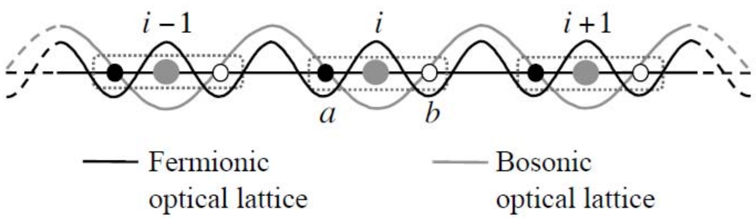
$$H = \int \Psi_{\sigma}^{\dagger}\left(-\nabla^{2} + V(r)\right)\Psi_{\sigma} + u\int \Phi_{\mu}^{\dagger}\Phi_{\sigma'}\Psi_{\sigma}^{\dagger}\Psi_{\sigma'} + v\int \Phi_{\sigma}^{\dagger}\Phi_{\sigma'}^{\dagger}\Phi_{\sigma'}\Phi_{\sigma} + \ldots$$

→ Use a superlattice



QUANTUM SIMULATION
HIGH ENERGY MODELS

- Matter + Gauge fields + Relativistic

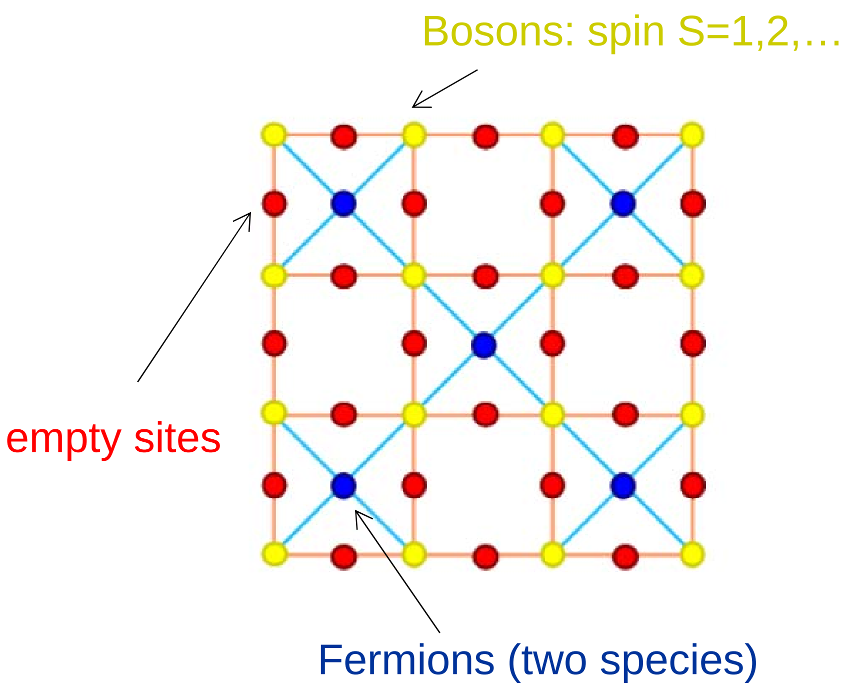


$$\frac{H_\Phi}{\hbar} = \int dx \left(v_s \bar{\Psi}_n \gamma_1 p \Psi_n + g m \Phi \bar{\Psi}_n \Psi_n + \frac{m^2}{2} \Phi^2 \right).$$

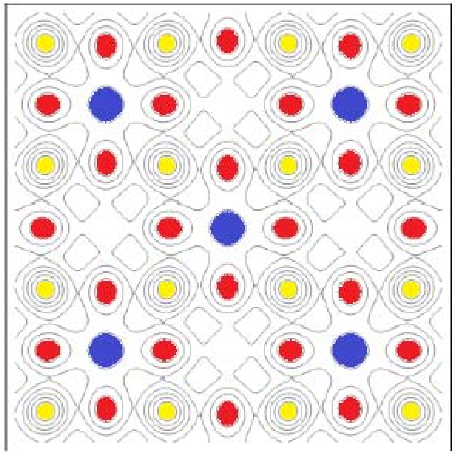
(Yukawa theory with infinite mass fields)

IC, Maraner, and Pachos, PRL 105, 1904´03 (2010)

- Higher dimension + Gauss law:



Laser configuration (5 lasers)



- Boson fields are replaced by 1 atom with 2S+1 internal levels
- Gauss law is enforced by an energy penalty

- Full cQED (Kogut Susskind) Hamiltonian

Zohar, IC, Reznik, PRL 109, 125302 (2012)
Zohar, IC, Reznik, arXiv:1208.4299

FERMIONS

$$\psi_{\mathbf{n}} = \begin{pmatrix} c_{\mathbf{n}} \\ d_{\mathbf{n}} \end{pmatrix} \qquad Q_{\mathbf{n}} = \psi_{\mathbf{n}}^{\dagger}\psi_{\mathbf{n}} - 1$$

$$H_p^f = i\eta \sum_{\mathbf{n},k} \left(\psi_{\mathbf{n}}^{\dagger}\sigma_k\psi_{\mathbf{n}+\hat{\mathbf{k}}} - H.C.\right) + M\sum_{\mathbf{n}} \psi_{\mathbf{n}}^{\dagger}\sigma_z\psi_{\mathbf{n}}$$

BOSONS (spins)

$$H_p^b = \sum_{\mathbf{n},k} \left(\mu\left(L_{z,\mathbf{n}}^k\right)^2 + 2\beta L_{x,\mathbf{n}}^k\right) + \Omega \sum_{\langle i,j \rangle} \left(L_{x,i}L_{x,j} + L_{y,i}L_{y,j}\right)$$

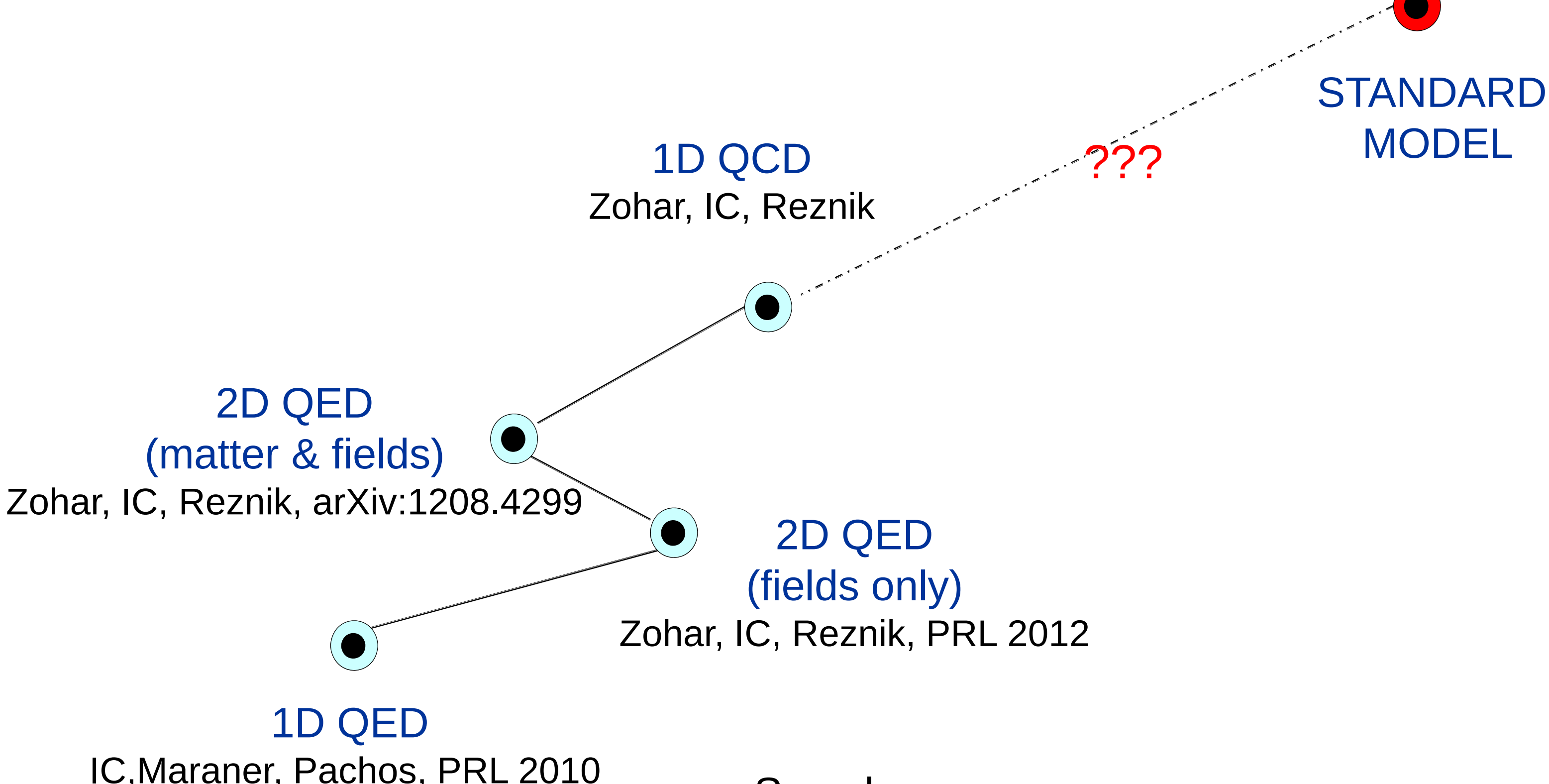
GAUSS LAW

$$G_{\mathbf{n}} = L_{z,\mathbf{n}}^1 + L_{z,\mathbf{n}}^2 + L_{z,\mathbf{n}-\hat{\mathbf{1}}}^1 + L_{z,\mathbf{n}-\hat{\mathbf{2}}}^2 - (-1)^{n_1+n_2} Q_{\mathbf{n}}$$

$$H_G = \lambda \sum_{\mathbf{n}} G_{\mathbf{n}}^2$$

CONCLUSION and OUTLOOK: Part II

ROAD MAP



See also:

- E. Kapit and E. Mueller, Phys. Rev. A **83**, 033625 (2011).
- D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, U.-J. Wiese, and P. Zoller, arXiv:1205.6366v1
- L. Tagliacozzo, A. Celi, A. Zamora, and M. Lewenstein, arXiv:1205.0496v1 [cond-mat.quant-gas].

QUANTUM MEMORY
SUMMARY

ROBUSTNESS

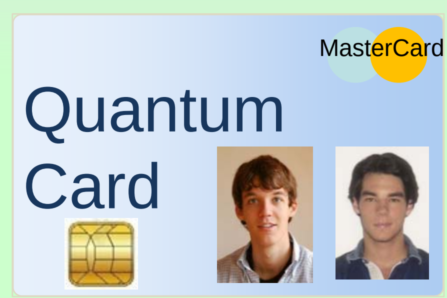
Depolarizing noise

- **No protecting** $T = O(1)$
- **Protecting** $T = O(\log N)$
- Solution: Dissipative protection

Hamiltonian perturbation

- **No decoding** $T = O(1)$
- **QECC in 2D/3D** $T = O(\log N)$
- **General recovery Majorana** $T = O(\exp N)$

APPLICATIONS: security proofs



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